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by

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ABSTRACT

Many statements have been made regarding the acceleration principle--its meaning, qualifications, applications and value. This thesis therefore is an attempt to note and summarize the main ideas surrounding the concept, and to place these in clear perspective.

The approach is thus to describe the nature of the acceleration principle, the forms it may assume and its limitations. As the main body of literature on the accelerator has been focused upon its value in explaining business cycles and economic growth, a synopsis of the more emminent views in these fields is undertaken.

It is concluded that this principle may best be considered as a partial theory of induced investment. However, problems of measurement make the accelerator a difficult concept to use for forecasting purposes or for planning economic development. Its major value is that, along with the multiplier, it provides a framework upon which a more comprehensive analysis of business fluctuations and growth may be constructed.

ACKNOWLEDGEMENTS

I would like to express my great appreciation for the assistance and encouragement given to me throughout the writing of this thesis by Professor W.D. Gainer. I am also very much indebted to Professor E.J. Hanson.

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TABLE OF CONTENTS

TABLES

FIGURES

CHAPTER I

INTRODUCTION

In this book, Business Cycles,[1] W.C.Mitchell noted that during the trade cycle, raw materials and producer goods in general varied more sharply both as to price and physical volume of sales than did consumer goods, that inventories were larger in booms than in hard times, and that raw material prices seemed to drop before the prices of finished goods did. Mitchell thus assumed that because the peaks and downturns of the producer good industries preceded the peaks and downturns of the consumer goods industries, that changes in the former must be the initiating factor in business cycles.

The interest of J.M.Clark was aroused by these studies and the conclusions reached. As a result, he reinterpreted the facts to show that changes in consumer purchases were actually the initiating factor causing the downturn in producer goods even though the latter occurred prior to the downturn in consumer buying. This came about because, as he saw it, the level of net investment was determined by the rate of change of consumer goods output. With an increasing rate of increase in such output, additional investment would be stimulated, and if the rate of increase should slacken off, net investment would decline absolutely. This concept, stated here in general terms, has come to be known as the acceleration principle.

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1. Mitchell, W.C., Business Cycles (Berkeley: University of California Press, 1913), pp. 99-103, 500-502.

Since 1917, when Clark first presented this idea,[2] much has been written about it.[3] In the past quarter century the acceleration principle has assumed an important place in the literature of the business cycle, and more recently, in growth models and the interaction of cycles and growth trends. (And for a short time, it was discussed in relation to the theory of pump-priming).

But there has been considerable criticism of the concept and many factors have been cited as qualifying or restricting its operation. Statistical testing has also been done in an effort to either prove or disprove it. The conclusions reached vary. Some writers have used it in its rigid form as a prime determinant in their theories of fluctuations or growth. Others have discarded it as being of no validity. Still others have struck a happy medium between these two extremes by utilizing the principle in a modified or qualified form.

The purpose of this thesis is to examine the nature of this concept, the forms it has taken, the factors which may affect

2. Clark, J.M., "Business Acceleration and the Law of Demand: A Technical Factor in Economic Cycles", Journal of Political Economy, Vol. XXV (March, 1917), pp.217-235 and reprinted in American Economics Association, Readings in Business Cycle Theory (Philadelphia: Blakiston Company, 1944), pp. 235-260.

3. Earlier references were made to this principle by such as Carver in 1903, Aftalion in 1909 and Bickerdike in 1914. The viewpoints of these men will be mentioned briefly in Chapter IV. But it was Clark's exposition on which most subsequent literature was based.

affect it and the manner in which it has been used in business cycles and growth theory. Chapter II will be devoted to a discussion of the nature of the acceleration principle and the type of interpretations it has been given when utilized as a rigid relationship between the rate of change of output and investment in capital goods.

Chapter III involves an outline of the many factors which have been cited from time to time as qualifying or limiting the operation of the principle, and the direction in which these influences may work. A brief review of the history of the theorizing about the place of the accelerator in business cycles is given in Chapter IV. One or two of the more recent models using the accelerator will be examined. The emphasis will be on the form the acceleration principle has taken and its importance in these models rather than on the intricacies of the models themselves. The profits principle will also be examined and its relationship to the acceleration concept pointed out. A few notes on the accelerator in the theory of pump priming will conclude the chapter.

Chapter V will be devoted to an investigation of the place of the accelerator in the theory of economic growth. This will involve examining the Harrod-Domar growth models and the question of the stability of these models. The capital coefficient will be compared to the acceleration coefficient. As with the previous chapter on business cycles, one or two of the recent models of economic growth will be examined, primarily to determine the place

assigned to the acceleration concept. The closing chapter will be an attempt to draw conclusions from the material in the foregoing chapters and to assess the importance of the accelerator in current theory and practice.

The field covered is broad. Consequently, the discussions on any one of the areas of economic theory mentioned will not be exhaustive. Instead, they will be illustrative--in an attempt to demonstrate first, the nature and limitations of the accelerator, and secondly, that whether one recognizes it as a valid and useful concept depends largely upon whether one interprets it in a broad or narrow manner and whether one expects it to be a definitive theory of investment or simply one of the factors which may or may not influence the volume of investment.

CHAPTER II

EXPLANATION OF THE ACCELERATOR

This chapter is an analysis of the meaning of the acceleration principle. It includes a discussion of its operation as a relation between changes in the quantity of consumer goods demanded and in the resulting changes in the amount of durable producer goods required. Its operation as a relationship between consumer services and durable consumer goods, and between consumer goods and inventories will also be noted. The magnitude of the acceleration effect as determined by the ratio of absolute changes in consumer sales and producer goods, will be examined and compared with the results when percentage changes are used. The accelerator will also be related directly to the theory of the firm. The longer range operation of this principle will also be discussed. Finally, a tabulation of the more common formulations of the acceleration principle will be made.

The Accelerator and Durable Capital Goods

It was as a relationship between changes in the quantity of consumer goods demanded and the magnified effect on the amount of durable capital goods required that the acceleration principle was first formulated. Table I presents an illustration of this type of relationship.[1] It is assumed that inventories are non-existent and that there are no

1. This example is adapted from R.A.Gordon, Business Fluctuations (New York: Harper, 1952), pp. 107-110. It should be noted that throughout the balance of this paper, whenever changes in demand are referred to they should be considered as changes in the quantity demanded unless it is otherwise stated.

changes in the amount of goods on order for future delivery.[2] Thus, it is possible to speak of changes in either sales or output of consumer goods since both will be the same. Also, prices are assumed to be unchanging so that changes in dollar values are deemed to reflect accurately changes in physical volumes.

Initially there is a periodic output of a consumer good of $100: total capital equipment required to produce this output is valued at $500, each unit having a life of ten years; these machines are assumed to have been installed evenly over a continuous period of time so that now, exactly ten per cent of them or $50 worth require replacing each year.

In the second period, output or sales rise by five percent from $100 to $105. But for this to occur, the number of machines producing them must also be increased by five per cent or by $25. Thus, a $5 increase in demand for consumer goods has resulted in a $25 increase in demand for capital goods. The increase in output of the consumer good has been magnified five times in the increase in demand for the capital goods. The production of capital equipment in this period, (and it is assumed that such production can be completed within the period), rises from $50 to $75 which covers replacement plus net investment.

In the third period, it is assumed that output of

2. See A.D.Knox, "The Accelerator Principle and the Theory of Investment: A Survey", Economica, Vol. XIX (August, 1952), pp. 273-274.

TABLE I

ACCELERATION PRINCIPLE AND DURABLE CAPITAL GOODS

Period	Output of Consumer Goods	Total Stock of Capital Goods Required	Additions to Capacity Required (Net Investment)	Replacement Capital	Total Demand for New Capital per Period
1	100	500	0	50	50
2	105	525	25	50	75
3	115	575	50	50	100
4	120	600	25	50	75
5	122	610	10	50	60
6	120	600	-10	50	40
7	115	575	-25	50	25
8	105	525	-50	50	0
9	100	500	-25	50	25

consumer product rises at a faster rate, the absolute increase in this case being $10. This in turn necessitates a further rise in the stock of capital of $50. The total demand for capital for the period has risen from $75 in period two to $100. Thus, it is seen that with consumer goods output rising at an increasing rate, a further increase in investment results. And the magnitude of this increase in investment, both absolute and percentage-wise, is greater than the increase in the demand for consumer goods. (Note that even if output of finished goods had increased at the same absolute rate in period three as in period two, i.e., at a rate of $5. total net investment would have at least remained constant).

In the fourth period, output of consumer product rises again, but instead of by $10, only by $5. This means that the net addition to capital stock required is only $25 so total investment now drops to $75. for the period. Thus it is seen that although output of consumer goods continued to rise, the fact that it rose at a decreasing absolute rate resulted in an absolute decline in the demand for new capital goods, i.e., in net investment.

In period five, a still smaller rise in consumer goods output results in a further decline in net investment and with replacement investment constant at $50, total investment also declines.

In period six and thereafter, output actually declines absolutely with the result that the required stock of capital also drops. This means that not only is net

investment not required but the usual amount of replacement is not required either. In period eight, the decline in output is so great that the stock of capital required is reduced such that even replacement is not necessary. Gross investment is zero, net investment is negative and equal to the replacement investment no longer being undertaken.

This is the acceleration principle in operation. The demand for capital goods is derived from the output of consumer goods in such a way that the change in demand for capital goods is determined by the change in the rate of change of consumer goods output. Where the amount of consumer goods output is increasing at a constant absolute rate, there will be a constant amount of net investment required per period. But a fall in this absolute rate of increase of consumer goods output will result in a fall in the actual amount of net investment required to maintain capital stock at the necessary level.[3] As Clark puts it "...the velocity of output in the capital making industries depends, not on the velocity of output in the industries which use the capital to make goods for consumption, but on its acceleration."[4]

3. See F.M.Somers, Public Finance and National Income (Philadelphia: Blakiston, 1949), pp.70-71; also Gordon, pp.110; G. Haberler, Prosperity and Depression: A Theoretical Analysis of Cyclical Movements, 3rd ed. enlarged by Part III. (New York: United Nations, 1952), pp. 88-96; S. Kuznets, "Relation Between Capital Goods and Finished Products in the Business Cycle", Economic Essays in Honor of Wesley Clair Mitchell (New York: Columbia University Press, 1935), pp. 211; D. Hamberg, Business Cycles (New York: McMillan, 1951), pp.107; Clark, "Business Acceleration...," pp.217-235.

4. J.M.Clark, Studies in the Economics of Overland Costs (Chicago: University of Chicago Press, 1923), p. 390.)

A deceleration of consumption results in a fall in the velocity of investment.

This relationship between consumer demand and net investment can also be shown in diagrammatical form using an adaptation of a graph presented by Hansen.[5]

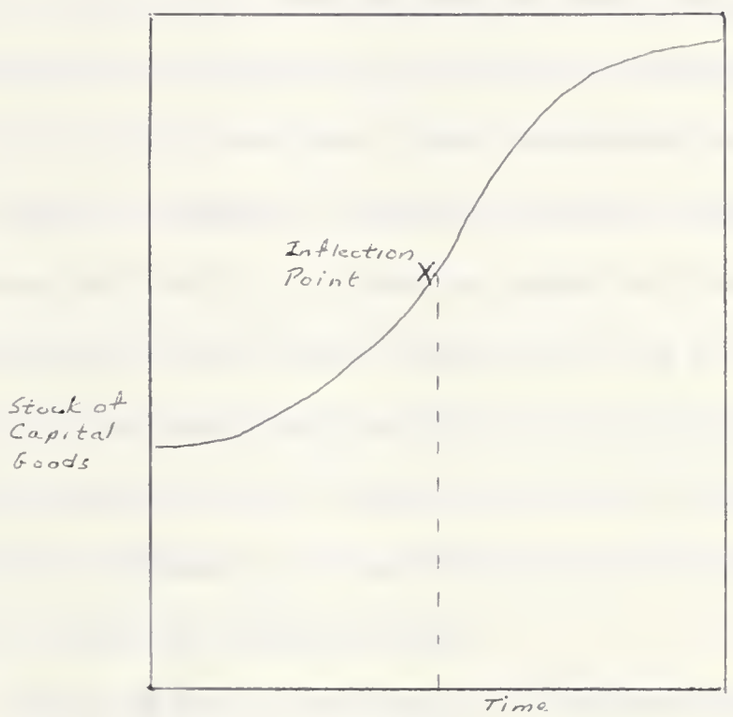


FIGURE 1a

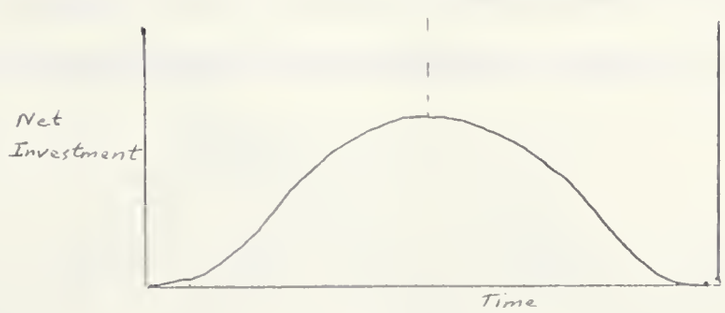


FIGURE 1b

5. A.H.Hansen, Business Cycles and National Income (New York: Norton, 1951), p. 181.

In this figure, it is assumed that consumer demand is rising, first at an increasing rate and then at a decreasing rate until it levels off again at a new higher level of demand. If a constant accelerator coefficient or ratio of capital stock to output is also assumed, then the stock of capital must rise in the same manner as the consumer demand. This is shown in figure 1a. From the capital stock requirements can be derived the increments of additional investment required, i.e., net investment; these increments are plotted in 1b. Thus two things are evident from these graphs. First, where the rate of increase in demand for consumer goods and hence for additional capital stock is at its peak, that is at X, net investment reaches its peak also. And secondly, where there is a decline in the rate of increase in demand for consumer goods and hence for more capital stock, net investment declines also.

Where there is oscillation in consumer demand rather than only a rise as shown above, then it is possible to show, as Hansen does,[6] both consumer sales and investment

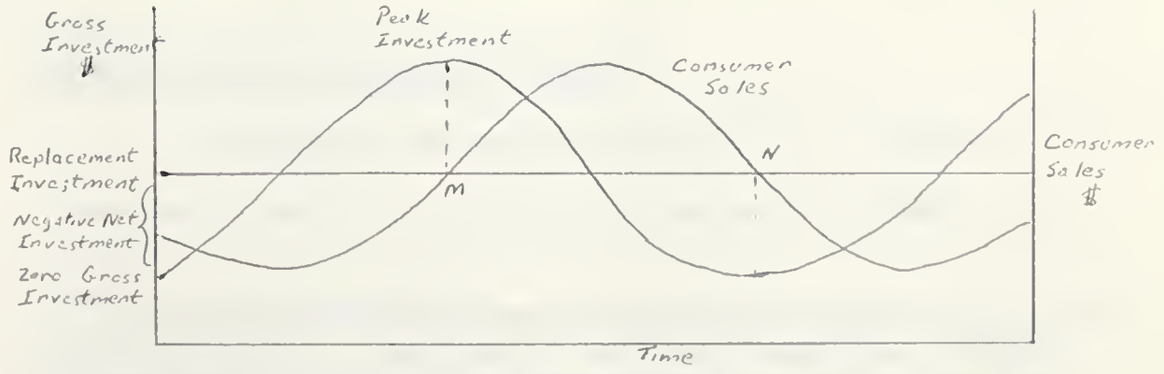


FIGURE 2

6. Ibid., p. 182.

in one graph. (See figure 2). This time replacement investment is also shown. Total investment is measured on the left side and consumer demand on the right hand side, each with their own scales. This figure is useful in portraying that as long as the rate of consumer demand is increasing, i.e., up to point M, that investment rises. Once consumer demand begins to increase at a decreasing rate, investment falls and continues to do so until consumer demand begins to decline at a decreasing rate as at N. At this point gross investment would begin to rise again.[7] In other words, it shows that the demand for producer goods varies not with the level of demand for the finished product but rather with the rate of increase of this demand. It also points out how the maximum demand for producer goods precedes the maximum point of demand for the consumption goods, i.e., it shows how the change in the direction of net investment, although it is caused by changes in consumer demand, actually precedes any change in the direction of consumer demand. ('Demand' is here being used as synonomous with sales or output.)

Absolute versus Percentage Changes

Throughout the analysis using Table 1, (page 7), a constant accelerator, or ratio of capital to output, of

7. The above diagram has been drawn showing that at the point N where the rate of decline in consumer demand has reached its maximum, that the gross investment required is just at the zero mark. This relationship will certainly not always hold in actual practice.

five was assumed. (This constancy is more or less true also of the original arithmetic examples of Aftalion, Bickerdike and Clark, although Clark did speak in terms of tendencies rather than stressing a constant proportionality of capital and output). The important point to note here is that this accelerator relates the absolute, not percentage increase in consumer goods output to the absolute amount of new capital equipment required.[8]

If the percentage increase in consumer goods to the percentage change in new capital equipment required, (for which some writers reserve the term 'magnification ratio'), had been related, a different ratio for each period would have resulted. For example, if Table 1 is again considered, it will be noted that in period two a five percent increase in output of consumer goods leads to a fifty percent increase in the output of producer goods; the magnification ratio is thus 50%/5% or 10. In the following period, an approximate ten per cent rise in output of consumer goods, (based on the output in period 2), leads to an increase in demand for new capital goods of from $75 to $100 or 33-1/3%; the magnification ratio would thus be about three and one-third. Thus when dealing in percentages, the relative size of the starting figures and the changes will affect the resulting ratios. A constant accelerator coefficient would no

8. See Gordon, pp. 110; Hamberg, pp. 105-108; Haberler, pp.90; A.H.Hansen, Full Recovery or Stagnation (New York: Norton, 1938), p. 48n. and Fiscal Policy and Business Cycles (New York; Norton, 1941), p. 364n.

longer obtain.

The size of the percentage magnification ratio is closely tied in with the durability of capital equipment.[9] Ceteris paribus the more durable the equipment and thus the lower the depreciation and replacement each period (assuming depreciation and replacement are the same) the greater will be the magnification effect on demand for capital goods of a given percentage increase in the demand for consumer goods. For example, if consumer demand rose by five per cent, then a similar increase would be required in the stock of capital. And if annual replacement of capital is also five per cent of the current stock (assuming durability of twenty years) then it means that the five per cent increase in consumer demand has led to a one hundred per cent increase in the demand for producer goods; the magnification ratio is twenty. But with the same percentage increase in consumer demand, if durability were only 10 years and thus replacement was ten per cent per year, there would only be a fifty per cent increase in the demand for capital, or a magnification ratio of ten.

Although this percentage magnification ratio is

9. See Haberler, p. 91; Gordon, pp. 109-110; Hamber, pp. 108-109; Knox, p. 272. Also of interest is W.J.Baumol's article, "Acceleration Without Magnification," American Economic Review, Vol. XLXI (June, 1956), pp. 409 - 412, in which he points out that the acceleration principle, although consistent with percentage and absolute magnification effects on investment, does not imply these will always occur. Depending upon such factors as the ratio of capital required per unit of output, and the phase of the business cycle, there may or may not be a magnified effect.

valuable in indicating how a percentage change in demand for consumer goods results in a much greater percentage change in production of producer goods, and is thus of great importance in business cycle analysis, it should be kept separate from the acceleration coefficient. Some writers do not make this entirely clear.[10] Others such as Gordon, Haberler, Knox, Kuznets and Somers do make the distinction.[11]

The Accelerator and Consumer Durables

The acceleration principle also holds true for durable consumer goods. Table 2 is presented as an example.[12] Here, the relation is one between the consumer demand for the services which houses provide, (which may be measured by the monthly rental value), and the demand for new houses as measured

10. For example, see Hamberg, pp. 105-106; A.W. Stonier and D.C. Hague, A Textbook of Economic Theory (London: Longmans Green, 1958), p. 428. Even Frisch's final formulation of the acceleration principle was partly expressed in terms of percentages: 'A decline in the rate of increase of Consumer-Taking will call forth an absolute decline in the demand for capital goods when and only when the percentage with which the growth rate of Consumer-Taking diminishes per year is larger than the percentage with which the capital goods are worn out per year.' "Capital Production and Consumer-Taking--A Rejoinder," Journal of Political Economy, Vol. XL (April, 1932), pp. 254. Clark also spoke in terms of a percentage. In one of his later statements he said that "... the rate of increase of demand must never shrink by more than one-half of one per cent." Capital Production and Consumer-Taking-- A Further Word," Journal of Political Economy, Vol. XL (October, 1932) pp. 691.

11. Gordon, p. 109; Haberler, p. 90; Knox, pp. 271-272; Kuznets, pp. 225-226; Somers, pp. 70-71.

12. This is taken from Gordon, pp. 108-110.

by their construction value. Thus, in period two, with an increase of one hundred families, the services of one hundred new homes are required; the demand for housing services has risen by $600 per unit times 100 or $60,000 which requires one hundred new homes worth $6,000 each or $6,000,000. (The acceleration ratio here is ten, since it requires a $6,000 home to provide services renting at $600 per period).

The replacement demand for homes, (which are assumed to have a durability of twenty-five years), is forty units per period or $240,000. The increased demand means that total construction must rise to 140 units or by $600,000. In period three where the rate of increase in demand for housing services rises again, an even larger total construction of homes is required. However, in period four when the rate of increase in demand for housing slackens off, so does the new housing constructed with the result that there is an absolute decline in total construction.

The Accelerator and Inventories

This same principle may also be illustrated with respect to inventories.[13]

13. Ibid., p. 110.

TABLE 2

ACCELERATION PRINCIPLE AND DURABLE CONSUMER GOODS

Period	No. of Families	Annual Rental Value of Housing (a) Units Required (Thousands)	Value of Total Units Required (b) (Thousands)	Replacement Demand		New Construction Required		Total Construction	
				Units	Value (b) (Thousands)	Units	Value (Thousands)	Units(c)	Value (b) (Thousands)
1	1,000	$ 600	$ 6,000	40	$ 240	0	0	40	$ 240
2	1,100	660	6,600	40	240	100	600	140	840
3	1,300	780	7,800	40	240	200	1,200	240	1,440
4	1,400	840	8,400	40	240	100	600	140	840
5	1,450	870	8,700	40	240	50	300	90	540
6	1,500	900	9,000	40	240	50	300	90	540
7	1,500	900	9,000	40	240	0	0	40	240

(a) Assumes unchanging rental value of $600 per period for each unit.

(b) Assumes unchanging original cost of $6,000 per dwelling unit.

(c) Equals increase in number of families plus replacement demand.

THE ACCELERATOR AND INVENTORIES

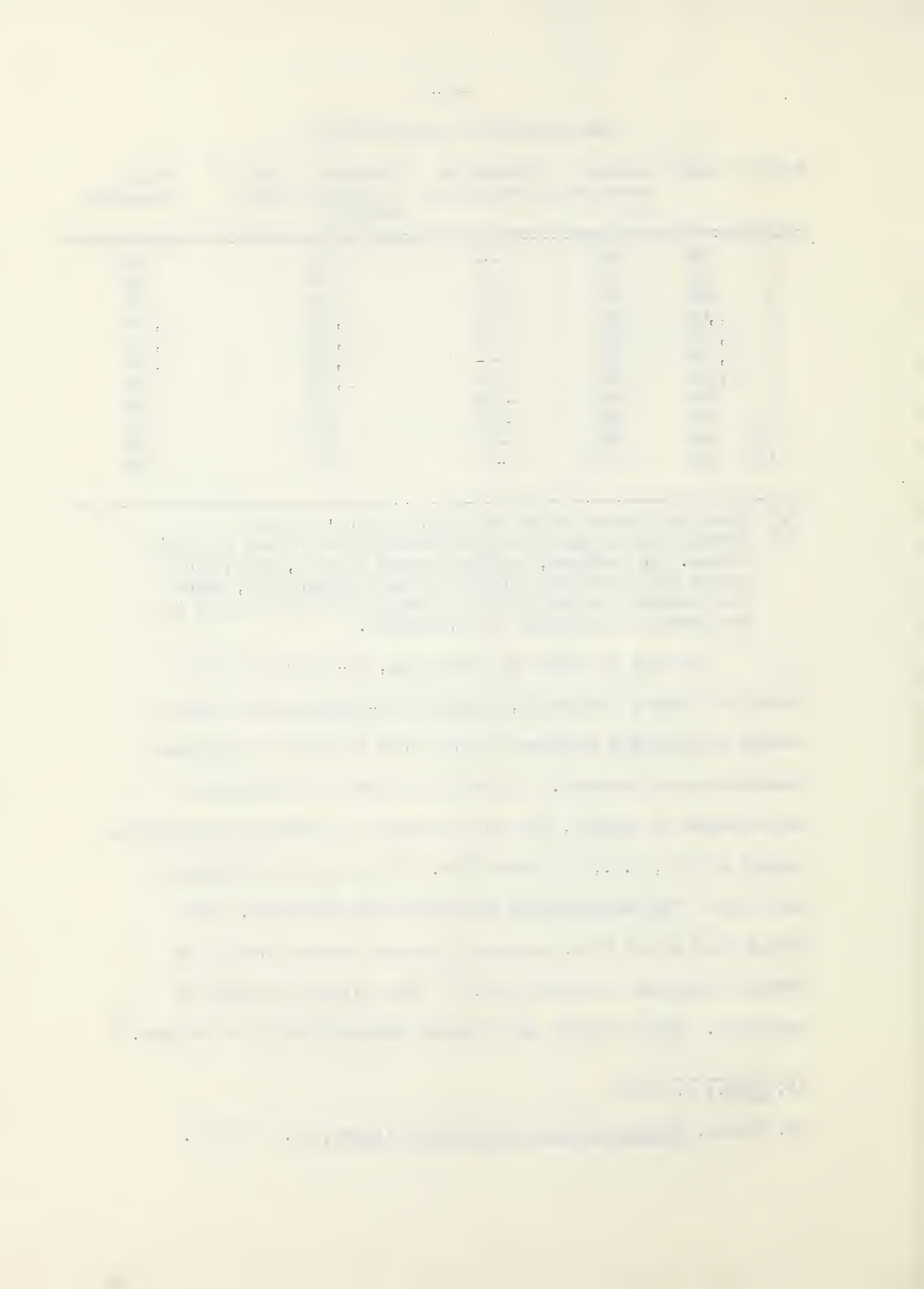
Period	Sales	Required Inventories (a)	Change in Inventories	Purchases to Replace Goods Sold (b)	Total Purchases
1	500	250	---	500	500
2	600	300	50	600	650
3	800	400	100	800	900
4	1,000	500	100	1,000	1,000
5	1,100	550	50	1,100	1,150
6	1,100	550	---	1,100	1,100
7	1,000	500	-50	1,000	950
8	800	400	-100	800	700
9	600	300	-100	600	500
10	500	250	-50	500	450
11	500	250	---	500	500

(a) These are assumed to be 50% of one period's sales
(b) Inventories and purchases are assumed to be valued at sales prices. In practice, they are valued at cost, which would reduce the acceleration effect which is shown here, since the acceleration ratio relates change in value of sales to the change in the value of inventories.

As long as sales are increasing, re-orders will rise relatively faster than sales, since the re-orders must be large enough to cover the increase in goods sold as well as the higher inventories now required. As soon as the rate of increase in sales begins to decline, the net investment in additional inventories begins to fall, i.e., in period five. It is also interesting to note that: "On the downswing when sales stop declining, even though they do not rise, purchases increase because there is no further reduction in inventories."[14] This situation occurs in period 11. These effects can be shown diagrammatically as follows.[15]

14. Ibid., p. 112

15. Hansen, Business Cycles and National Income, pp. 184-186.



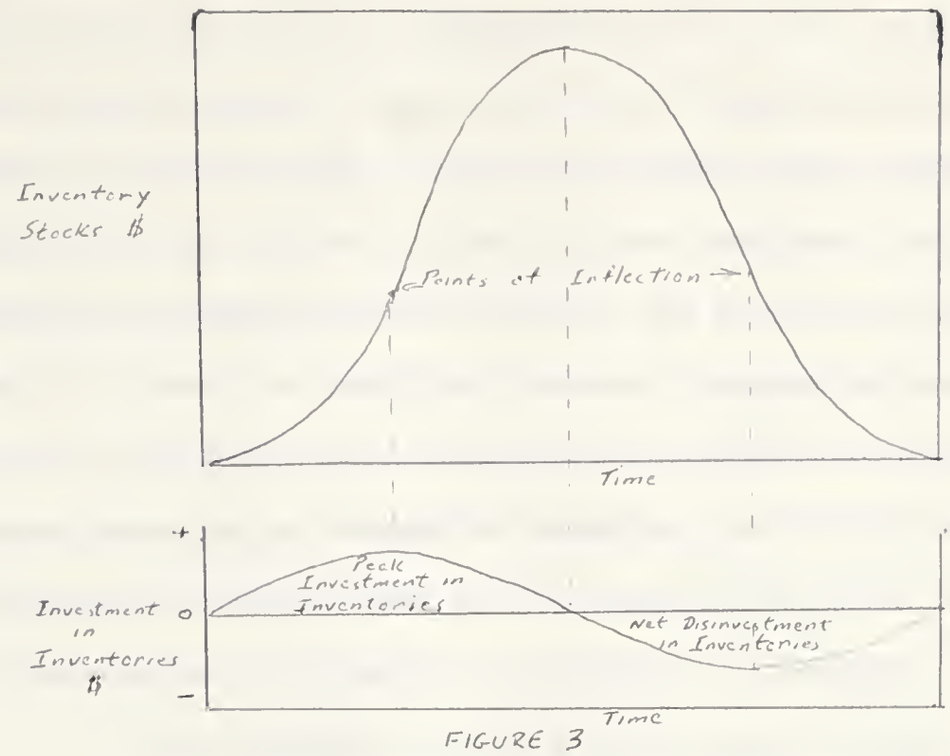


FIGURE 3

This example, assumes that inventories fluctuate along with sales, the two reaching high and low points together. (Time lags are disregarded). The relation between fluctuations in inventories stocks (controlled by sales volume) and net investment (and disinvestment) is as indicated in figure 3. As long as sales volume and thus inventory stocks continue to rise at an increasing rate, net investment will continue to rise, and when sales and stocks required begin to increase at a decreasing rate, net investment begins to decline.

The Accelerator in the Theory of the Firm

For a clearer understanding of the acceleration principle, it is useful to approach it from the viewpoint of the theory of

the firm.[16] It will be assumed that a single firm is involved which moves smoothly from one position of equilibrium to another with perfect foresight, there are no restrictions on financing expansion, the factors of production are homogenous and perfectly divisible, expectations are accurate, the firm has no excess capacity, there are additional factors of production available for hire, there are just two factors of production--capital and labor, there are no changes in technology, and finally that long run analysis is being used so that the firm can alter the amounts of both factors in response to a change in its output.[17]

The isoquants or equal product lines in Figure 4 are the curved lines convex to the origin. They show the various combinations of capital and labor which can be used to produce the level of outputs x, x+1, and x+2, etc. The lighter, straight lines represent the various amounts of capital and labor which can be purchased with expenditures of e_0, e_1 and e_2, etc. (Being straight, they

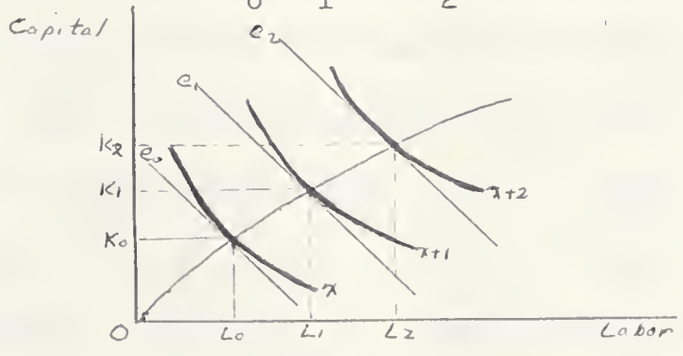


FIGURE 4

16. This analysis is adapted from R.S.Eckaus, "The Acceleration Principle Reconsidered", Quarterly Journal of Economics, Vol.LXVII (May, 1953), pp. 209-230.

17. The usual assumptions such as that firm seek to maximize their profits and that perfect knowledge of prices, costs and opportunities prevails are also taken for granted here.

imply pure competition in the purchasing of factors and that factor prices are constant for this analysis. But actually they could be slightly concave to the origin indicating some degree of monopsony; but in this case the lines as drawn above would be the slopes at the points of tendency, the only condition being that these be a unique tangency position for each expenditure line.)

Under these circumstances if the firm wishes to produce x units it will combine capital and labor in the proportions K_o and L_o. And if it wishes to increase output to x + 1 units it will combine the factors in proportion K_1 and L_1, i.e. it will move from one point of tangency between the equal product curve x and equal expenditure line e_o, to the tangency point of x + 1 and e_1, it being well known from the theory of the firm that the cheapest combination of factors is where the marginal significance of the one factor, in this case capital, in terms of the other, labor, equals the relative money prices of the two factors which occurs at these points of tangency.

From this the accelerator principle can be demonstrated. If the firm expands from output x to x + 1 or Δx_1, it uses $K_o K_1$ or ΔK_1 of capital and L_o L_1 or ΔL_1 of labor. Now, expressing the increase in capital, ΔK_1, as some factor B, times Δx_1 the increase in output, the accelerator principle simply stated is:

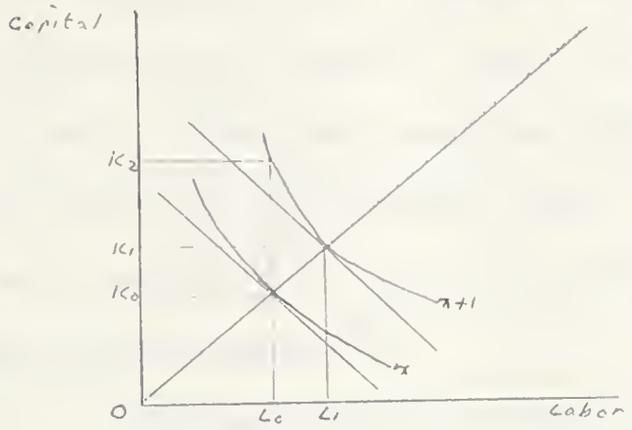
$\Delta K_1 = B_1 . \Delta x_1$ where B_1 = the accelerator coefficient.

And if Δx_1 is a unit change in output, then

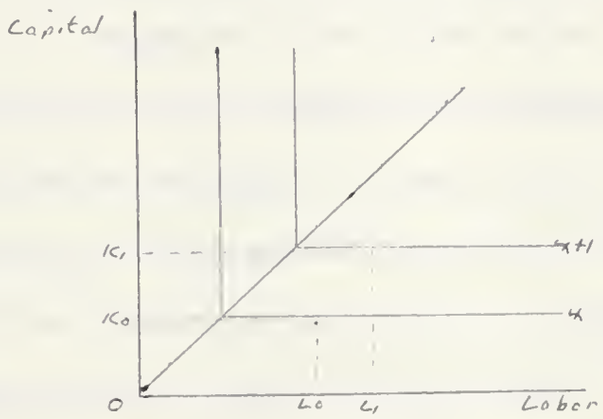
$\Delta K_1 = B_1$ i.e., the number of additional units of capital

required for an additional unit of output = the accelerator.[18]

18. It is important to note that the accelerator is the ratio of the increase in capital stock over the increase in output when all other factors are increased at the same time. This relationship is to be distinguished from the reciprocal of the marginal productivity of capital which is the ratio of the increase in capital to the increase in output when all other factors are held constant. This can be easily illustrated with an isoquant map and with assumptions similar to those used above. In the increase of output from x to x+1 both labor and capital were increased so that the accelerator was $K_o K_1 / \Delta x_1$. But if labor was held constant at L_o units, and x+1 units were produced by increasing capital only, this would mean $K_o K_2$ units of capital would be required. In this case the marginal productivity of capital would be $\Delta x_1 / K_o K_2$ and its reciprocal is $K_o K_2 / \Delta x_1$, which is a larger value than the accelerator.



These two values would only be the same if the marginal productivity of other factors was already zero so that the only increase in total product that occurs when capital and other factors are both increased would be due to the extra capital. This is illustrated in the isoquant opposite where labor and capital are now non-substitutable. If output is x, when K_o units of capital and L_o units of labor are employed by the firm, labor is obviously redundant and its marginal net product is zero. When capital is increased to K_1 then output is raised to x+1, regardless of whether labor is left at L_o or increased to L_1. The change in output is due to the capital alone. The accelerator coefficient and the reciprocal of the marginal productivity of capital are the same.



If the firm should expand to x + 2 units of output it would involve more capital and labor; the accelerator principle could then be represented as

$\Delta K_2 = B_2 \cdot \Delta x_2$ = the accelerator coefficient.

Since these accelerator coefficients link together both the outputs which are flows per unit of time (and whose size depends on the time unit chosen) with stocks of capital which are not flows, their magnitude depend on the time units chosen; the longer the time unit chosen, the larger the output can be produced with a given capital stock and thus the smaller the accelerator.[19]

The analysis reveals that in only two situations will the accelerator coefficient be constant: one case is where the factor price ratios remain constant and there are constant returns to scale; thus a proportional increase in inputs will produce a proportional rise in output. The production function would be homogenous of the first degree. The expansion path of the firm would be a straight line passing through the origin as indicated in Figure 5 below. If this same graph were viewed from the side with capital along the vertical axis and output along the horizontal axis (labor not being visible from this angle), it would appear as in Figure 6.

The acceleration coefficient for the increase in output from x to x + 1 would be the tangent of the angle BAC or $\frac{BC}{AC}$. The coefficient for the increase from x + 1 to x + 2 would

19. Kuznets, pp. 215-217.

be $\frac{DE}{BE}$ which is equal to $\frac{BC}{AC}$.

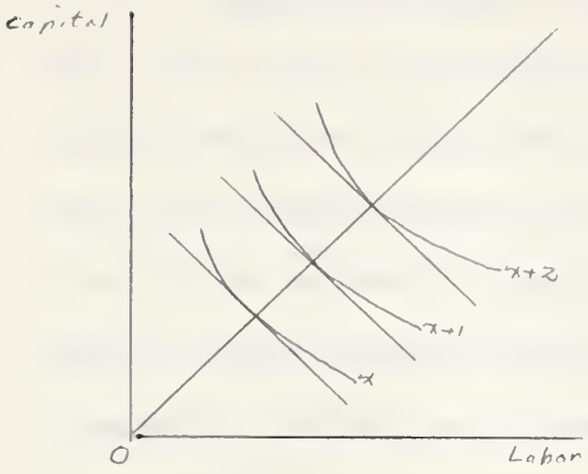


FIGURE 5

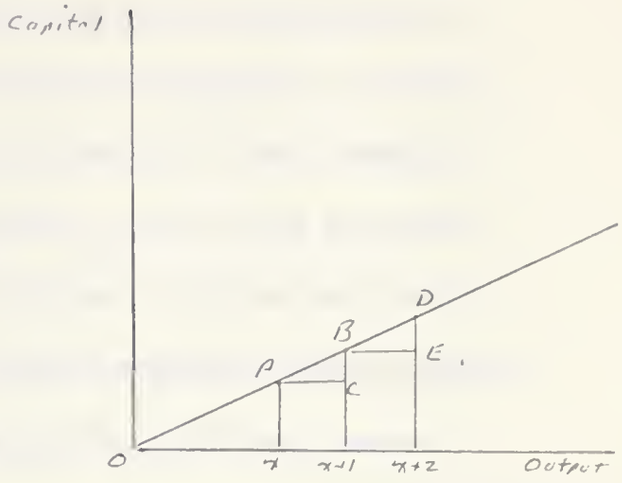


FIGURE 6

The second situation where a constant accelerator would occur is when constant coefficients of production exist along with constant returns to scale. Then, even if factor price ratios did vary, or the firm exercised some degree of monopsony power, factors would be combined in constant proportions and would produce proportional increases in productions.[20]

In all other cases, it is quite likely that a variable accelerator would exist. And if the simplifying assumptions made at the beginning of this analysis such as indivisibility of capital equipment, credit availability etc. are dropped, it becomes even more unlikely that the firm will have a constant accelerator except over small ranges of output. These possibilities are discussed in the following chapter.

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20. In the short run, when the firm cannot change its capital stock in response to an increase in output, the acceleration principle would not operate.

The Longer Run Acceleration Principle

Another approach to the acceleration principle is that of the rough relationship between the growth of industry and the demand for its product or services.[21] For example, when a new industry begins to develop, such as the railways after 1850 or automobile after World War I, there may come a period of rapid growth of demand which requires great expansion of capacity; this may be followed by a period when demand expands but at a less rapid pace, so that new additions to capacity are smaller; and finally comes a period of relative stability of output where the present capacity is sufficient to meet current needs. "No further net investment, over and above replacement would take place until required by new technological changes." Eventual decline would be the final step in this process.[22]

The time period involved here is longer than the "long run" spoken of up to this point. It is not just a period

21. Gordon, pp. 104-106. Gordon is one of the few writers to make such a clear cut distinction between the acceleration principle as it operates in the normal 8-10 year business cycle and its operation in the growth and decline of industries as a whole. However this possibility was mentioned as early as 1917 by Clark in his original article, although there he used it not as a special type of acceleration but simply to clarify the point he was making regarding the relationship of output to investment. See "Business Acceleration..." pp.223-224. Tinbergen also hints at this approach in "Statistical Evidence on the Acceleration Principle, "Economica, Vol. V (May, 1938), p. 166. Hansen makes a much clearer statement of it in Fiscal Policy and Business Cycles", 1941, p. 283.

22. Gordon, P. 105.

long enough for a firm to enlarge or reduce capacity in the face of changes in its demand over the course of a business cycle, but instead may extend over several business cycles. It could perhaps be compared to the time period as suggested by the Kondratieff long wave.

The relationship between output and net investment under these circumstances is certainly not a rigid one, and it is subject to the qualifications that will be discussed in the next chapter; nevertheless, it is useful concept to keep in mind.

Various Formulations of the Acceleration Principle

A number of equations have been used to represent the acceleration principle in its basic form. These have varied according to whether the accelerator is viewed as a micro or macro economic concept, whether only consumer goods or all final goods are considered, and whether lags are provided for. A representative list of these has been compiled here as an indication of the variety of interpretations that have been given to this principle even in its basic form.

(1). The Accelerator as a Relationship between Consumer Goods and Producer Goods.

The simple relationship between changes in the rate of change of consumer goods output of a firm (or sales, if inventories and orders on the books are disregarded) and net investment in additional capital stock by a firm, as discussed in the early pages of this chapter, is usually expressed in the following manner:

$I(n)=A[O(n)-O(n-1)]$, where $I(n)$= net investment by the firm in period n, and $O(n)$ and $O(n-1)$= consumer goods output by the firm in periods n and n-1 respectively; thus $O(n)-O(n-1)$= the increase in output between periods n and n-1; A=the accelerator co-efficient.[23] (The length of the period is here assumed to be long enough that investment projects undertaken may be completed in the same period.) This formulation could also be used to express the relationship of demand for consumer services and durable consumer goods.

(2). As a Relationship between and Two Successive Stages of Production

Many firms are not engaged in production of equipment with which to supply the consumer goods industries. Thus, the acceleration effect as spoken of above would exclude them. Consequently the principle has been considered as applying also to business firms whose investment decisions are geared, not to consumption but to the production of capital goods i.e., to the investment decisions of other business firms.[24] Kuznets brought this point out clearly in his statement:

The terms 'finished products' and 'capital goods' are used in the present discussion not in an absolute sense,

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23. This can easily be identified with the formulation $\Delta K_1 = B_1 \Delta X_1$ used when approaching the accelerator from the theory of the firm type of analysis: $\Delta K_1 = I(n)$= net investment; $B_1 = A$= the acceleration coefficient; and $\Delta X_1 = O(n) - O(n-1)$=the change in output.

24. See Somers p. 72.

> but as relative to each specific link in the buying-selling relations that run through the economic system. Capital goods are thus raw materials, machinery, buildings, and any other commodity which an entrepreneur needs to produce what from his standpoint is the finished product. What are finished products to a given entrepreneur may obviously be capital goods to another entrepreneur. Similarly, 'production' is used in the broadest sense, to include not only manufacturing and extracting industries, but also transportation, trade, and service.[25]

Wright brings this point out also when he states that the acceleration principle may depend upon the rate of increase of innovation.[26]

This step places the accelerator on a more practical level. The accelerator becomes the ratio between the amount of 'capital goods' produced in any stage of production and the change in production of 'finished' goods in the succeeding stage. This may be shown as: $O(n)^{m-1} = A[O(n)^{m} - O(n-1)^{m}]$ where $O(n)^{m}$ represents output of stage m in period n, $O(n-1)^{m}$ represents output of that stage in period n-1 and $O(n)^{m-1}$ is output of the preceding stage in period n. A is again the accelerator coefficient.

(3). As a Relationship between Aggregate Consumption and Aggregate Net Investment.

To obtain a representation of the acceleration principle on a macro economic basis, aggregate net investment

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25. Kuznets, p. 21ln.

26. D.McC. Wright, "A Neglected Approach to the Acceleration Principle, " Review of Economic Statistics, Vol. XXIII (May, 1941), pp. 101.

has on occasion, been shown as depending on aggregate consumption. This has been expressed as:

$I(n)=v[C(n)-C(n-1)]$, where In is aggregate investment in period n, C(n) and C(n-1) are aggregate consumption in periods n and n-1 respectively, and v is the acceleration coefficient applying to these aggregate concepts.[27] This is the type of assumption used by Samuelson in his interaction model of the accelerator and multiplier concepts which will be discussed in Chapter IV.

(4). As a Relationship between National Income and Aggregate Net Investment.

In recent years, the use of the acceleration principle as a relationship between national income (or output) and aggregate net investment has been the most common practice.[28] This approach assumes a certain normal ratio between the total stock of real capital in the economy, (measured in terms of money value), and the level of national output. The amount of capital entrepreneurs desire to have will depend on the level of demand for final output, which is, for the economy as a whole, determined by national income. Thus, the stock of capital in existence at any time will be a function of the level of national income.

27. One criticism of the acceleration principle which applies to this formulation alone will be pointed out in Chapter III, p. 54.

28. The following discussion is based on R.C.O. Matthews, The Trade Cycle (Digswell Place: James Nesbit, 1960), pp. 12-13.

And the rate of addition to this stock (net investment) will be a function of the rate of increase of national income. (Autonomous investment is ignored in this set of assumptions).

(a) Unlagged Form: This relationship is often represented as: $I(n)=v[Y(n) - Y(n-1)]$ where $I(n)$ is net investment in period n, $Y(n)$ and $Y(n-1)$ are national income in periods n and n-1 respectively, and v is the accelerator coefficient for the economy. Thus the amount of net investment done during the period n will be sufficient to raise the stock of capital by an amount equal to the increase in income that is expected to take place in the current period over the previous period, multiplied by the accelerator. This formulation assumes either that the total increase in demand which takes place in the period occurs at the beginning of the period so that entrepreneurs may undertake new investment on the basis of this actual increase, or else that the entrepreneur makes an estimate of the increase in income that may occur over the entire period and bases his investment actions on this estimate.

(b) One-period Lag: In this case the equation is usually expressed as: $I(n)=v[Y(n-1) - Y(n-2)]$. This approach assumes that the level of capital stock at the beginning of period n was sufficient to produce $Y(n-2)$ efficiently, i.e., that the capital was being worked at optimum efficiency when producing $Y(n-2)$.[29] Thus in period n, the amount of

29. The concept of an "optimum" level of output will be discussed in Chapter III when the effects of surplus capacity are being considered.

net investment undertaken will be sufficient to bring capital stock up to the level where output Y(n-1) may be produced efficiently. The entrepreneur is thus always just one period behind in bringing his capital stock up to the level required.

(c). Distributed Lag: The acceleration concept has also been represented as:

$$I(n)=v_1[Y(n-1) - Y(n-2)] + v_2[Y(n-2) - Y(n-3)] + \ldots$$

where $v_1 + v_2+\ldots= v$.[30]

This states that investment outlay in period n depends on output changes in a series of past periods. It is equivalent to saying that an output change in period n gives rise to investment changes in a series of future periods. This may come about because investment is composed of a "mixed bag" of inventories, machinery and plant. The investment in each of these may be undertaken in a different period. Furthermore, where payments for capital are spread out over several periods, the above equation would also apply.

Resume and Conclusion

The basic meaning of the acceleration principle is that investment depends upon the rate of change of output. This relationship may be conceived of as existing between output of consumer goods and investment in producer goods,

30. J.R.Hicks uses this approach. See "A Contribution to the Theory of the Trade Cycle (Oxford: Clarendon, 1950), p. 174.

between demand for consumer services and expenditure on durable consumer goods, between sales and changes in inventories, or between any two consecutive stages of production. Furthermore, it may be viewed as applying to a single firm's investment decisions; to the longer run growth, maturation, and decline of any industry; or to aggregate concepts of income and investment for the economy as a whole. In any of these situations investment may be considered as occurring in the same period as the change in output which induces it, or in some future period or periods. But in every case, the rigid relationship of investment and the rate of change of output remains.

CHAPTER III

QUALIFICATIONS OF THE ACCELERATION PRINCIPLE

Having now discussed the essential relationship involved in the acceleration principle, one is now in a position to examine the factors that have from time to time been presented as affecting or qualifying this relationship. In his original paper Clark discussed some of these such as surplus capacity, availability of funds, and the general effects of expectations.[1] Many writers since that time have elaborated on these as well as discussing a number of other influences. What will be attempted here is to outline these factors and the manner in which they may affect the acceleration principle. And at the same time the discussion will illustrate the diversity of opinions and interpretations that have been expressed.

Surplus Capacity

One of the first and most important criticisms that is usually made of the acceleration principle is that the presence or absence of excess capacity will alter its operation considerably. The argument is that if an industry or firm is operating at less than full capacity (which will often be the case in recessions as well as the early part of most upswings), an increase in sales (and therefore output--providing inventories are ignored) will not induce additional investment as the principle would suggest. Instead, existing

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1. J.M.Clark, "Business Acceleration..."

idle equipment can simply be utilized. The accelerator under such circumstances becomes non-operative. Only if, as Kuznets suggested,[2] it is the policy of a firm to maintain a certain level of excess capacity would an increase in sales cause additional investment. This would be done to restore the agreed upon level of idle capacity.

Although most writers on the principle have mentioned this matter, few have defined precisely what they mean by "full capacity". This is understandable, as it is an elusive concept. A review of some of the possible definitions and their implications is worthwhile.

One definition which has been used, usually implicitly, is: the level of output at which the firm's plant and equipment are being utilized as fully as possible; i.e., machines are operating at maximum speed for a full 168 hours a week, and no further addition of labor, or improvement in the quality of labor, administration, or raw materials will increase output.[3] Full capacity in this sense would rarely if ever be achieved.

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2. Kuznets, "Relation Between Capital Goods..." pp. 232-235, suggests this is the case particularly where consumer demand fluctuates under normal conditions such as a department store which will have surplus capacity except during peak periods such as Christmas and Easter. Then, the firm can, when demand rises, raise the rate of utilization of the equipment over short periods, cutting down costs per unit and increasing profits. He also cites an example how a firm may maintain excess capacity to meet peak periods up to where the additional profits from meeting this peak demand are just offset by the higher fixed costs per unit the rest of the time.

3.Knox, pp. 278-279; also see Tinbergen, "Annual Survey: Suggestions on Quantitative Business Cycle Theory", Econometrica, Vol. III (July, 1935), p. 253; Somers, p. 77; C.D.Long, Jr., Building Cycles and the Theory

But as it has been frequently implied as a starting assumption in discussions of the accelerator, it is interesting to examine its implications more closely. One of the standard expressions of the accelerator, viz: $I(n)=A[O(n-1) - O(n-2)]$, is normally taken as signifying that the increase in output in period n-1 over output of period n-2 will induce investment equal to a multiple, A, of this increase.[4] That is, consumer goods or final goods output rises first followed by a rise in net investment.[5]

However, if there is already a full capacity level of output as just defined, no expansion of output can ensue without first the production of additional capital equipment. The sequence of events is thus--a rise in demand for consumer goods which augments demand for capital equipment; an increase in output of capital equipment which may then be utilized

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3.Continued
of Investment (Princeton: University Press, 1940,) pp. 41,58; p. 51; J.M.Clark, "Additional Notes on Business Acceleration and the Law of Demand", Preface to Social Economics (New York: Farrar and Rinehart, 1936, and reprinted in American Economics Association, Readings in Business Cycle Theory (Philadelphis, Blakiston, 1944, pp. 256-257; and also Stonier and Hague, A Textbook of Economic Theory, pp. 430-431.

4. "Expectations" should be an integral part of this analysis, but the discussion of these are reserved for later in this chapter.

5. What is being discussed must be distinguished from that aspect of the accelerator principle which states that changes in the direction of investment precede changes in the direction of consumption spending; for example, even though consumption is still rising, a slowing down in its rate of increase is deemed to cause an absolute decline in investment.

to augment the output of consumer goods in response to the higher demand; i.e., the increase in investment precedes the rise in output of consumer goods rather than following it. At least one writer who has been aware that this sequence may occur, when capacity is defined as indicated above, denies that it should even be considered as an example of the acceleration principle in operation.[6]

However, if the principle is reinterpreted as a relationship between sales of final goods (rather than output) and investment, then, through a reduction in inventories, a rise in sales may occur first, thus preceding the change in investment. Then, too, if one takes the dollar volume of sales rather than real quantities, price increases due to the initial shortage of final goods will be tabulated as well; thus, although there may be little rise in real consumption of final goods, the dollar value may be up considerably. Again, if one counts the increases in unfilled orders on the order books as part of the increased sales, the usual sequence of the acceleration principle, with the expansion of final goods sales preceding the increase in investment, will be possible.

Another definition of full capacity, more popular in recent writings,[7] involves the idea of the "optimum" output

6. Eckaus, pp. 221.

7. It was discussed as early as 1923 by Clark in Studies in the Economics of Overhead Costs (Chicago; University of Chicago Press, 1923), p. 91; See also J.R.Hicks, A Contribution ...p. 39; Knox, p. 278; H. Neisser "Critical Notes on the Acceleration Principle", Quarterly Journal of Economics, Vol. LXVIII (May, 1954), pp. 253-274; and S.C.Tsiang, "The Accelerator, Theory of the Firm, and the Business Cycle", Quarterly Journal of Economics, Vol. LXV (August, 1951), pp. 328-331

of the firm, or the point of "optimum efficiency"; i.e., a firm would be producing at full capacity when it was producing at the optimum level for which it was designed. This concept in itself is vague and could easily be ambiguous. But if one associates it with the standard theory of the firm analysis, it becomes more manageable. The optimum level of output could then be interpreted as the minimum point on the firm's short run average total cost curve, (assuming this curve is U-shaped.)[8] This is shown in the figure below. SAC is the firm's short run average total cost curve, SMC the short run marginal cost curve, LAC the long run average total cost, and LMC the long run marginal cost curve. It is assumed for simplicity that there are no internal economies or diseconomies of scale, and thus LAC=LMC. MR, is the initial marginal revenue curve of the firm. As it is drawn, the firm is producing output OM, where

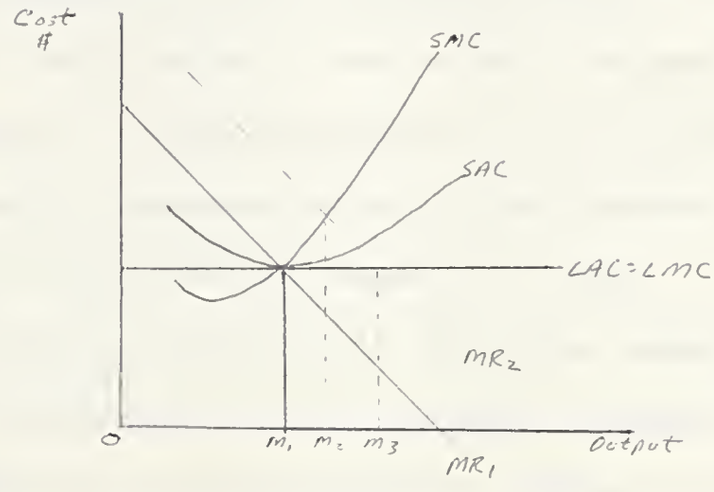

FIGURE 7

8. The assumptions underlying the U-shaped short run average total cost curve are outlined in any standard book on price theory; eg., Stonier and Hague, A Textbook..., pp. 107-116.

MR=MC which is also the point where SAC is at its minimum; i.e., the firm is operating at "full capacity." (The average revenue curve is not shown as it is not essential to the analysis and would only complicate the diagram).

If an increase in demand occurs, the sequence of events can be easily traced. Assume the rise in demand results in the marginal revenue curve shifting to MR_2. The firm's response, assuming that it seeks to maximize profits, will be to increase output along its SMC to where MC again equals MR_2 at output OM_2. No addition to capital stock has yet been made. But, if it is assumed that his rise in demand will be permanent, the firm will expand capacity along its LMC curve, i.e., net investment takes place; the firm can now produce OM_3 units of output. In brief, the sequence is--an increase in demand, a rise in output along the short run marginal cost curve, net investment which thus increases capacity, and finally a further rise in output. Thus, as with the previous definition of capacity, the usual statement of the acceleration principle where output increases, followed by an increase in investment, does not explain the full sequence of events. If one interprets this principle as applying to the initial expansion of output along the SMC curve and the subsequent enlargment of capital facilities, while disregarding the additional increase in output of final goods which may occur once the plant is enlarged, it would seem to eliminate this problem.[9] However, there remains

9. Hicks in A Contribution... used this approach on the assumption that "...an increase in output does not follow as a necessary consequence of an increase in capacity." p. 40n.

another sticky point: the accelerator co-efficient which will now be the ratio of the change in investment to the initial change in output, will seldom if ever be of a constant value over any length of time. Consequently, its effects in the upswing of the business cycle will be quite unpredictable and thus it becomes difficult to use it in combination with the multiplier concept (as indicated in the following chapter) as a valid explanation of business cycles. Its magnitude will depend upon the elasticities of the short run and long run marginal cost curves as well as the slope of the new marginal revenue curve which results from an increase of demand.[10] (This will readily be seen if one refers to figure 7. The initial enlargement of output from OM_1 to OM_2 will vary with the elasticity of SMC. Similarly, the subsequent expansion of plant to where MR_2= LMC will depend on the slope of the MR_2 curve and the elasticity of the LMC curve.)

This discussion illustrated the variety of interpretations that may be given to the acceleration principle. It also indicates part of the reason why it is most difficult to obtain any real agreement as to the validity and usefulness of this concept.

Asymmetry

Another common criticism of the acceleration principle is that it is asymmetrical. In the downswing of the business cycle, the acceleration effect is limited by the extent of

10. Tsiang, pp. 330.

of depreciation. One of the most concise and frequently quoted statements on this point is that by Tinbergen:

> Very strong decreases in consumer's goods production must not occur. If the principle were right, they would lead to a corresponding disinvestment and this can only take place to the extent of replacement. If annual replacement amounts to 10 per cent of the stock of capital goods, then a larger decrease in this stock than 10 per cent per annum is impossible. A decrease could not lead to a 15 per cent decrease in physical capital as the acceleration principle would require. It is interesting that this limit is the sharper the greater the duration of life of the capital goods considered.[11]

This point is generally accepted.

Expectations

Whether a lagged or unlagged equation is used to represent the acceleration principle, it is generally assumed that the change in output between whatever two periods the entrepreneur is using to determine his investment actions, will be permanent. The importance of this assumption rests on the fact that capital equipment is generally durable and must yield satisfactory returns over a period of years.[12] However, expectations may not necessarily be of this type. As Haberler says:

> ...a whole scale of more or less optimistic anticipations should be distinguished, ranging from the expectation that the increase in demand is only temporary, via the

11. Tinbergen, "Statistical Evidence..." p. 165; Clark, in Business Acceleration..." mentioned that investment could not be reduced as quickly as it can be increased but did not carry the statement any further.

12. Hamberg, Business Cycles, p. 114.

> expectation that it will be maintained to the expectation that it will go on rising at a constant or increasing rate. In other words, there are various ways in which an actual increase can be extrapolated in to the future.[13]

In an attempt to provide for the effects of such divergence of expectations, Metzler developed the "coefficent of expectations".[14] [15] This approach has since been utilized by several other writers so as to give expectations explicit recognition in the acceleration equation.[16] As an example, the tack followed by Dernberg and Mcdougall is presented: The coefficient of expectations, γ, is assumed to be some percentage of the observed change between output in periods n-1 and n-2. At the beginning of period n, firms already

13. Haberler, p. 306n

14. L.A.Metzler, "The nature and Stability of Inventory Cycles", Review of Economics and Statistics, Vol. XXIII (February, 1941), pp. 113-129.

15. Long, in Business Cycles...used the concepts of neutral, positive and negative confidence. Hicks, in Value and Capital (Oxford: Clarendon, 1939),developed the idea of an elasticity of expectations in relation to price, pp. 205. It was defined as "...the ratio of the proportional rise in expected future prices...to the proportional rise in current price". It has been suggested that this could be interpreted as a quantity elasticity of expectations relating to changes in output. (eg. Haberler, p. 306). However the idea has not been fully expanded. Even Hicks does not try to use it in his later book, A Contribution to the Theory of the Trade Cycle. Metzler, however, in "The Nature and..." did relate the concept to specific time periods and attempted to equate it with his own coefficient of expectations.

16. Eckaus, pp. 222-223 and T.F.Dernberg and D.M.McDougall, Macro Economics, The Measurement, Analysis, and Control of Aggregate Economic Activity (New York: McGraw-Hill, 1960) pp. 227-228.

have sufficient capital to produce the output of period n-1 efficiently, i.e., there is no investment lag; induced investment occurs in the same period as the rise in income which necessitates it. Thus, $I(n)=v[Y(n) - Y(n-1)]$. (1). The expected expansion of output of period n over n-1 is assumed to be equal to the increase of output of period n-1 over n-2 multiplied by γ, the coefficient of expectations. In other words, $Y(n) - Y(n-1)=\gamma[Y(n-1)-Y(n-2)]$. Thus by substituting for $Y(n) - Y(n-1)$ in (1) the result is $In=v\gamma[Y(n-1) - Y(n-2)]$.[17] The effective accelerator becomes $v\gamma$. Where $\gamma=1$, it indicates that the rise of Y(n) over Y(n-1) is expected to be the same absolute amount as the enlargement of Y(n-1) over Y(n-2); If $\gamma = 0$, entrepreneurs expect that there will be no increase in output in period n over n-1 and consequently, regardless of how great the accretion in output of n-1 over n-2, no induced investment will be forthcoming in period n. The effective accelerator becomes zero. If γ is negative, output in period n is expected to drop below the output of period n-1. Thus the effective accelerator is negative; disinvestment or negative net investment will take place in period n.

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17. It is worth emphasizing that this equation arrived at by Dernberg and Mcdougall is not the same as $I(n)=v[Y(n-1)-Y(n-2)]$ mentioned in Chapter II, with simply an extra coefficient, γ, introduced. $I(n)=v[Y(n-1) - Y(n-2)]$ assumed that investment in period n will be made to bring capital stock up to the level required to produce output Y(n-1) efficiently; whereas, $I(n)=v\gamma[Y(n-1) - Y(n-2)]$ assumes that investment in period n will be made to being capital stock up to the level required to produce Y(n) efficiently, where Y(n) is estimated as being equal to Y(n-1) plus some percentage, γ, of the increase in output of period Y(n-1) over Y(n-2).

This type of approach accomplishes its objective in making expectations an explicit part of the accelerator equation. But it does little to clarify the operation of this principle. For as Long succinctly states:

> ...we find that the psychology of expectations is essentially unstable due to the reciprocally corroborative quality of the unorganized investment crowd; that it often, if only temporarily, interacts cumulatively with the real factors; that, depending upon the type and source of the expectation, it may either bring about or prevent the event expected; that it is not susceptible to quantitative measurements; and that its quality and direction may be judged only by knowledge of the 'preconceptions' of the interpreters.[18]

Thus until a workable theory relating to expectations can be developed, their effects on the acceleration principle must be left somewhat vague.[19]

In concluding this section, one or two illustrations may be given of the differences in interpretation that have been placed on the accelerator as a result of expectations being introduced. The accelerator has, on occasion been deemed as operative where a rise in investment takes place as a result of a rise in the rate of increase of expectations by entrepreneurs in a consumer or other final goods industry.[20] In this situation, it is neither output or sales which induce investment, but expectations alone.

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18. Long, p. 53.

19. See R.M.Bissell, "The Rate of Interest", American Economic Review Supplement, Vol. XX VIII (March, 1938), p. 34.

20. Wright, p. 100.

In another instance, it was suggested that if the acceleration principle included expectations, that even what is normally considered as autonomous investment due to innovations need not necessarily be considered as outside its scope.[21] That is, to a certain extent at least, innovations are made in anticipation of an expected rise in the rate of consumer demand for the product involved. However, few would wish to stretch the accelerator this far.

Replacement Investment

Replacement of capital equipment may be necessary either because of age and obsolescence taking their toll, or because of wear and tear as in the case of machinery and direct use as with raw materials. Although replacement has been a common subject of discussion in relation to the accelerator, some writers have developed their arguments using only one of the above determinants to the exclusion of the other.[22] If

21. Somers p. 85; also see R.F.Harrod, The Trade Cycle; An Essay (Oxford: Clarendon, 1936), p. 83, and Hansen's criticism Full Recovery of Stagnation (New York: Norton, 1938), p. 51.

22. For instance, A.C.Pigou, Industrial Fluctuations (London, Macmillan, 1929), p. 110; R. Frisch, "The Interrelation Between Capital Production and Consumer-taking", Journal of Political Economy, Vol. XXXIX (October, 1931), pp. 646-654, Clark, "Business Acceleration..."; and Hansen, Fiscal Policy and Business Cycles, p. 278 all assumed only the second reason was involved. Kuznets assumed the second reason applied to inventories of middlemen while the first reason applies to durable equipment, p. 219n3. Knox, considers both, p.288-289.

both are considered, the formulation of the acceleration principle in terms of gross investment may be as follows:

$$G(n)=v[Y(n) - Y(n-1)] + B.Y(n-1) + D.K.(n-1),$$ [23]

where G(n) is gross investment in period n (excluding autonomous investment), v[Y(n) - Y(n-1)] is the standard expression for the accelerator, Y(n-1) is output in period n-1, B is the relation between the level of output and replacement required, B.Y(n-1) is thus the replacement necessitated because of wear and tear, etc., K(n-1) is the stock of capital at time n-1, D is the relation between this stock and replacement expenditures at time n, and D.K(n-1) is replacement required due to age and obsolescence.

The importance of replacement investment was first brought to the fore in the Clark-Frisch controversy,[24]. Clark originally related the change in consumption to gross investment, and considered the replacement portion as a minor factor which would not greatly affect the operation of the acceleration concept.[25] Frisch, however, pointed out that even if there were a decline in the rate of increase in output, gross investment

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23. Adapted from Somers p. 76n.

24. Clark, "Capital Production and Consumer-taking--A Reply", Journal of Political Economy, Vol. XXXIX (December, 1931), pp. 814-816; and "Capital Production and Consumer-taking--A Further Word", Vol.XL (October, 1932), pp. 601 - 603; Frisch, "The Interrelation Between Capital Production.." pp. 646-654; and "Capital Production and Consumer-taking--A Rejoinder", Journal of Political Economy, Vol. XL (April, 1932), pp. 253-255; also "Capital Production and Consumer-taking--A Final Word", Vol. XL (October, 1932), p. 694.

may not decline but that instead, a rise in replacement investment could well be sufficient to offset the decrease in net investment. He outlined a situation where total investment would approach asymptotically some level and remain there; thus the acceleration effect relating to limited decreases in the rate of increase in consumer goods output would not be in evidence for gross investment as a whole.

More recent analysis indicates that one can neither assume that gross investment will approach such a level nor will it necessarily respond as the acceleration principle suggests. Too many unpredictable influences are involved.

First of all, as Kuznets states, the life period of productive equipment is subject to wide dispersion and the longer the average period of life of equipment, the larger (in absolute time units) will likely be the variations in the actual life of single productive units of this type.[26]

Furthermore, even if one accepts the statement that

> Assuming that firms seek to maximize profits, there is an incentive to replace when the present value of the anticipated stream of profits from the new machines minus that from the old is greater than the price of the new machine minus the 'scrap value' of the old...[27]

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25. Clark, "Business Acceleration...", pp. 236-238.

26. Kuznets, p. 237

27. Knox, p. 285. In summary form, replacement will occur when $R_{1t} - R_{2t} > P_{1t} - S_{2t}$ where t is the time period, subscript 1 is new machines, subscript 2 is old machines, R is aggregate profit, P is Purchase price of new machines and installation costs, and S is scrap value of the old machines.

the final answer as to when restoration of capital will occur and how it will affect the accelerator is still not easily determined. Constant changes in technology make it difficult to predict accurately the expected life of new machines; for example, it may pay to ignore the first technically superior machine that comes on the market and wait for the next one, if further developments are imminent. Then again, within many firms, there may be no standard approach for calculating when replacement should take place, i.e., no one constantly comparing costs of operation and returns of new machines developed against existing equipment. Thus replacement may depend much on mere chance.[28] Nor can it be definitely said at what stage in the business cycle replacement becomes profitable.[29] Varying expectations which will affect the marginal efficiency of capital also enter in here.[30] In short, the timing on restoration expenditures involves a complexity of considerations much beyond the simple allowance for age and intensity of use as suggested in the early part of this discussion.

Under these circumstances, should the acceleration principle be applied only to net investment? The problem, from a statistical verification point of view, then becomes one of trying to separate net investment from replacement

28. Ibid., p. 286-288.

29. Ibid., p. 286-289. Matthews, pp. 62-66 also given an excellent discussion of this matter.

30. Hamberg, Business Cycles, p. 116.

expenditures. Usually, firms replacing equipment install something larger or better, i.e., replacement often involves improvement.[31] The question of how to treat differences between depreciation reserves and actual replacement enters as well. Harrod suggested that expenditure above depreciation should be considered as net investment, and those below as disinvestment.[32]

Clearly then although replacement investment must be considered when discussing the acceleration principle, its nature is such that if incorporated, the accelerator may not be evident in its effect on gross investment, and yet to exclude it is next to impossible. In other words, its presence makes statistical verification of the principle extremely difficult and leaves the matter open for dispute as to whether the accelerator is a valid concept or not.

Availability of Funds.

The supply of funds available for expansion, has been a conventional part of the dialectics on the accelerator. First thoughts may be that a single firm operating in a competitive capital market would face a perfectly elastic supply curve for funds as its actions would have no effect on the ruling interest rate.

But as Tsiang states, "...the marginal cost of borrowing subjective as well as objective, will rise rapidly

31. Harrod, The Trade Cycle: An Essay, pp. 54-55.

32. Ibid.

as the firm approaches the limit of its borrowing capacity."[33] For instance, lenders will rely on the net value of a business firm for their security and will not be willing to lend any company more than up to a given percentage of its share or equity capital. Then to, Tsiang states:

> ...there is an increasing subjective risk accompanying the increase of indebtedness relative to the firms own capital, because the greater the amount of borrowed capital relative to the firm's equity capital, the greater the danger of the entrepreneur's income from his own capital being wiped out when the average rate of profit falls short of the rate of interest on its loans and the greater the danger of bankruptcy in case of liquidation.[34]

In addition, the possibility of raising new share capital is not unlimited in a short period of time. Generally speaking, the larger the volume of new shares a firm attempts to place on the capital market within a given period of time the lower will tend to be the price it must accept from them.[35]

Probably the major source of funds available to a company is from its own reserves, or savings. If it is assumed that demand for final goods is expanding rapidly, yet for any number of a variety of reasons,[36] savings are limited,

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33. Tsiang, p. 232. Clark also mentioned, "Business Acceleration..." p. 231-232.

34. Ibid., p. 333.

35. See Clark, "Business Acceleration..." p. 252; Kuznets, p. 227; Haberler, p. 101, 309; J.S. Duesenberry, Business Cycles and Economic Growth (New York: McGraw-Hill, 1958), pp. 39-44, emphasizes the supply of funds problem as the major limitation on the acceleration principle.

36. Somers, pp. 86-90.

this fact coupled with the limitations on funds suggested above will tend to diminish the acceleration effect. Any discussion of savings should also consider profits. The relation of profits to investment however, has frequently been considered as a separate theory of investment; for this reason, further comment on this matter is left for another section of this chapter.

Shortage of Other Factors

In addition to a possible insufficiency of funds, other bottlenecks such as labor shortages, union restrictions on productivity, and strikes may hinder the working of the acceleration principle.[37] But such limitations are not necessarily used as arguments against the principle's validity. Instead, it has been suggested that supply inelasticites in the investment goods industries may produce a cumulative downswing in which the acceleration principle plays an important part. Matthews summarizes this point of view in a concise manner:

> Bottlenecks due to the inelastic supplies of factors are likely to be met at different stages of the upswing in different sectors of the economy. A hypothesis based on a particular application of this is that the ceiling that halts the boom is not general full employment but full employment of the investment industries. Since investment fluctuates more violently than consumption, the pressure on resources may be expected to be felt in the investment industries at an earlier stage of the upswing than elsewhere in the economy. The possibility of transferring labour from other

37. Hamberg, Business Cycles, p. 384; also cf. Long, p. 44-45.

> industries is limited, especially in the short period. This bottleneck will slow down or halt the expansion of investment. This will lead through the multiplier to a corresponding slowing down in the expansion of national income, and this in turn, if it is sufficiently pronounced will lead through the capital stock adjustment principle to an actual contraction of investment.[38]

The cumulative downswing will thus get under way; the accelerator in this case is an important part of the mechanism causing the downswing.

Technological Change

Constantly changing technology will, as indicated above, affect the timing on replacement of equipment through its influence on the obsolescence rate of such capital. Consequently the acceleration effect on gross investment may be altered.

It was also seen that if expectations are introduced, autonomous investment such as may result from new technological developments, could conceivably be considered as coming within the ambit of the acceleration principle.

Apart from these possibilities, discussions of the effects of technology changes are frequently confined to the manner in which the acceleration coefficient may be altered. Technology may be neutral, capital-using, or capital-saving. Neutral technology reduces labor and capital per unit of output

38. Matthews, pp. 154-155. He uses the term "capital stock adjustment principle" which as will be seen in the subsequent chapter, is no more than a generalization of the acceleration concept.

in the same proportions.[39] Its effect will be ceteris paribus, to lower the ratio of capital to output and thus the accelerator coefficient. A capital-saving innovation[40] reduces capital per unit of output and thus would also decrease the accelerator coefficient. A capital-using innovation increases the capital required per unit of output and thus would enlarge the accelerator.

Some attempts have been made to demonstrate how forces operating within the economy will determine the type of technology applied in production which will thus alter the accelerator. Haberler, for instance, suggests that variations in the going interest rate, wage rate, and general out-look as to the behavior of these and other cost items in the future may affect the type of machines produced.[41] To illustrate, if interest rates were thought to be higher than normal then this, ceteris paribus, may tend to shorten the "round-about" method of production (if there is a choice) and cheaper, less durable equipment may therefore be produced.

A more ambitious claim for the effects of changes

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39. Taken from Joan Robinson, The Rate of Interest and Other Essays (London: Macmillan, 1952), pp. 40, 48-51. Other definitions have been given by R.F.Harrod, Towards a Dynamic Economics (London, Macmillan, 1948), pp. 22-27; and J.R.Hicks, The Theory of Wages(London: Macmillan, 1932), pp. 121-127.

40. A distinction is generally made between "invention" which relates to the actual discovery of the technology, and "innovation", which refers to the introduction of the new development to operating practice. But this difference is not of importance to the present discussion.

41. Haberler, p. 97, 307.

in techniques was expounded by Hayek.[42] He suggested that not only would the accelerator coefficient be altered but that this in turn would produce the cyclical fluctuations of the economy. At full employment, if rigid interest rates and wages as well as immobility of labor are assumed, then the increasing demand for goods would raise their prices and hence profits. The techniques using less capital would have the highest profit per unit of capital; this would encourage a shift to such methods and thus produce a lower accelerator coefficient. This would tend to reduce the absolute volume of investment and cause a downturn in economic activity. Similar forces operating in the opposite direction were deemed to initiate the upturn.[43]

As with the other qualifications of the accelerator principle, it can be seen that the views on the effects of technological change are far from unanimous.

Indivisibility of Capital

> "If net investment is to be strictly a function of the rate of growth of output, then the units into which the stock of capital equipment is divisible must be the same as those into which output is divisible.

These words of A.D.Knox,[44] concisely summarize another

42. F.A.Hayek, Profits Interest and Investment (London: Routledge, 1939), Part I; also The Pure Theory of Capital (London: Macmillan, 1941) pp. 377-96.

43. For a good review and criticism of this approach, see Somers pp. 93-101 and the references cited there.

44. Knox, p. 280.

limitation of the rigid acceleration principle. It would be the rare occurrence if capital equipment were divisible as suggested above; new equipment may be bulky and only justified if output has risen considerably. In addition there may be lulls in investment (particularly if the investment is of any size) because new investment may interfere indirectly with current output through a division of managerial energies.[45]

The Macro-Accelerator

On a macro-economic basis, the accelerator has, as mentioned in chapter two, been considered both as a relationship between aggregate consumption and aggregate investment, and also as a relationship between national income and aggregate investment. Use of the former relationship will likely result in a distorted measurement of the acceleration effect. To illustrate, an increase in consumption may require more machinery which in turn might necessitate augmenting the output of the iron and steel industries. This may further expand the need for machinery--and so on. The aggregate increase in investment will thus be much larger than if the change in consumption had been related only to the succeeding stage of production. To interpret the acceleration principle in this manner would mean that alterations in investment would depend more on the structure of production and relationships existing between the various stages of production than on the technical relationship which lies

45. Ibid.

at the basis of this concept.[46]

Most recent formulations of the macro-accelerator avoid this difficulty by relating aggregate income to investment. This makes changes in investment a function of both changes in consumption and investment. But it may be argued that even this approach, because it involves a summation of the acceleration effects for all individual firms each with diverse expectations, credit facilities etc. will not produce a constant acceleration effect for any degree of time; and if it did it would be merely by chance.[47]

In some cases, an increase in investment may occur even though there was no expansion of output in the previous period over output of the period prior to that. The accelerator may therefore be deemed to be an unsatisfactory explaination of investment behaviour in this case. However, those who defend the principle may argue that although there has been no increase in aggregate demand, a shift in demand has occurred so that there has been a rise in the rate of increase of demand facing those industries which require more capital per unit of output and this shift is what has stimulated investment.[48]

Summary

The foregoing discussion has pointed out the various

46. Bissell, pp. 31-32; Somers, pp. 90-91.

47. Somers p. 91-92. Also cf. Tinbergen, "Statistical Evidence on..." pp. 166-167.

48. Hamberg, Business Cycles, pp. 112-113

factors which from time to time, have been mentioned as qualifying the operation of the acceleration principle. Moreover, it illustrated the difference of opinions that have occurred with respect to most of these influences. However, these many opinions tend to cloud the picture rather than clarify it. It will therefore be worthwhile to gather the main arguments together so that they may be viewed as a unit.

If one assumes that a given increase in the rate of change of output occurs, then, the rigid accelerator should produce a certain enlargement of net investment. However, there may already exist excess capacity in the producer goods industry so that no further investment is necessary; there would be no acceleration effect. Even if there were no excess capacity (when 'full capacity' is defined as the minimum point on the firm's short run average total cost curve), the change in investment which will result will not bear any constant relationship to the initial change in output which occurs as the firm moves to the right along its short run marginal cost curve. The extent of the induced investment will depend upon the elasticities of the short run and long run marginal cost curves as well as the slope of the new marginal revenue curve.

Then again, the change in the rate of change of output may be expected to be only temporary; in this event no induced investment may occur. On the other hand, if expectations are that not only will the given change of output be permanent, but that output will increase even more in the future, then, the investment induced may be greater than that required to

meet the rise in output; it may be sufficient to increase capacity so as to meet expected future expansion of demand; i.e. investment may be based on the rate of increase of expectations rather than just on the rate of change of actual output.

Moreover, if a firm faces a shortage of funds either because of lack of internal savings or inability to float additional shares or bonds, the enlargement of investment which the increase of output calls for under the acceleration principle may be reduced. Similarly, a shortage of other factors of production such as skilled labor, or competent managerial assistance may also hamper enlargement of plant size in the face of rising output of finished goods.

Technological changes, by altering the amount of capital required for a given amount of output, may also alter the amount of additional investment undertaken when demand rises. Also, since most capital equipment is indivisible, new investment will not proceed smoothly as the rate of change of output rises. Rather, it will occur in lumps.

These are the type of influences which will augment, mitigate, or even wipe out the acceleration effect of increases in the rate of increase of final goods output. When the rate of increase of output begins to decline, so that as a result of the accelerator, investment also tends to decrease, then some of these factors such as expectations, or technological change may again alter the acceleration effect. But now, in addition, replacement investment may obscure the acceleration effect on total investment. For, if as a result of the

previous rise in investment additional replacement is now required, then this increase in replacement may offset any decline in induced investment so that there is no obvious change in total investment. But, it is difficult to exclude replacement because it usually involves improvement or larger equipment etc.

Once output declines absolutely, as in the downswing of the business cycle, the acceleration effect will be limited to the size of depreciation charges, i.e., disinvestment can only occur up to the extent of depreciation rather than by the full amount which may be required by the decline in output. These, then, are the essential qualifications of the acceleration principle, when it is viewed as a micro economic concept.

When the macro approach is used, the above qualifications will continue to affect the accelerator. But now, since all firms and industries each with their own expectations, credit facilities etc. must be aggregated, it means that a constant acceleration effect will be a more unlikely possibility, than even where this concept in considered on a micro basis. With these limitations of the accelerator in mind, one must turn to the results of empirical studies that have been designed to test this concept.

Empirical Results

Statistical studies conducted to test the accelerator have been somewhat inconclusive. J.M.Clark's examination of the relationship between purchases of rolling stock and

railway traffic over a period of 15 years were favorable.[49] Tinbergen's imposing study of rolling stock purchases and railway traffic in the United Kingdon, France and Germany as well as a similar type of analysis on the cotton spinning and shipping industries in the U.K. gave little support to the acceleration principle.[50] As an alternative he submitted the 'profits principle' as an explanation of induced investment.[51] The studies of Kuznets, also of the railway industry, over the period 1891-1930, were, like Tinbergen's, unfavorable.[52] Subsequent testing by such as Wilson, Hultgren, Manne, Chenery, and Hansen have produced mixed results.[53] From the above discussions of the numerous possible qualifications of the accelerator, these phenomena are about what one would expect; the

49. Clark, "Business Acceleration..." pp. 245-249

50. Tinbergen, "Statistical Evidence on..." pp. 168-176. Somers strongly questions the approach and statistical methods used by Tinbergen, and thus doubts his conclusions.

51. More will be said on this subject in the section which follows.

52. Kuznets, p. 246-267.

53. T. Wilson, Fluctuations in Income and Employment (London, 1948), pp. 114ff; T. Hultgren, American Transportation in Prosperity and Depression (New York: National Bureau of Economic Research, 1948), pp. 157-169; A.S.Manne, "Some Notes on the Accelerator Principle", Review of Economic Statistics, Vol. XXVII (May, 1945), pp. 93-99; H.B.Chenery, "Overcapacity and the Acceleration Principle", Econometrica, Vol. XX (January, 1952), pp. 1-29, and Hansen, Full Recovery or Stagnation?, pp. 50, 281, and Fiscal Policy and Business Cycles, p. 49.

principle is in evidence, given certain definitions of concepts, but certainly not all the time--other factors may nullify or at least mitigate its effects.

The Profits Principle

The profits principle is another widely accepted explanation of induced investment. A brief review of the opinions on this concept as expressed by two or three leading authors will serve to illustrate its basic nature.

Tinbergen, for example, states that

> Whether or not an entrepreneur decides to invest depends first of all on whether he expects to make profits or not. Therefore, the number of entrepreneurs planning investment will depend on profit expactations.[54]

The factors he considers as usually exerting the greatest influence on profit expectations are:

(1) the magnitude of currently earned profits;
(2) the price of capital goods; and
(3) the rate of interest.[55]

Other authors, he adds, prefer to use profit margins, i.e., the margin between average selling price and average prime cost, as the chief explanatory factor, instead of total profits. However, he embraces total profits as being more reliable. In some cases, Tinbergen uses the profit rate, i.e., profit as a percentage of capital; but on the whole, he believes that

54. Statistical Testing of Business Cycle Theories (Geneva: League of Nations), Vol. I, p. 34. Also see J. Tinbergen and J.J.Polak, The Dynamics of Business Cycles (London, 1950), ch. 13.

55. Statistical Testing... p. 36.

"...the results will...be very nearly the same..." [56] as where total profits are used. Profits also facilitate the financing of new investment expenditures both by making more funds available directly, and by improving the possibilities of raising additional capital through security issues or loans.

Kalecki also recognizes this latter influence of profits. As he states,

> Investment decisions are closely related to 'internal' accumulation of capital, i.e. to the gross savings of firms. There will be a tendency to use these savings for investment, and, in addition, investment may be financed by new outside funds on the strength of the accumulation of entrepreneurial capital.[57]

But unlike Tinbergen, he considers the rate of profit to be more important than the level of profits. For if the effect of the level of profits is rigidly applied, then providing profits remain at a given level, net investment should also remain constant. But in fact this will not necessarily happen; if a high, constant level of profits continues for several periods, investment may well decrease since a large number of the investment projects that were made possible by the high profits will be undertaken in the first year and could not be duplicated in the years following.[58] He takes into consideration the increases in the volume of capital stock which, when the level of profits is constant, will mean

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56. Ibid., p. 37

57. Theory of Economic Dynamics (London: Allen and Unwin, 1954), p. 97.

58. "A New Approach to the Problem of Business Cycles", Review of Economic Studies, Vol. XVII (1949-1950), p. 61.

> ...a reduction in the rate of profit....[which] tends to restrict the boundaries of investment plans. This effect is most easily seen in the case where new enterprises enter the field and thereby render investment plans of the established firms less attractive.[59]

Knox also endorses the rate of profits as a more important determinant of investment than the level of profits.[60] The latter, he feels, through providing additional funds, exercises more of a permissive influence on investment both as to its amount and the timing of it. He adds however, that such higher profits may exert a more positive influence than this "...in so far as the availability of funds stimulates some inquiry into the openings for investment..."[61]

Some writers are less explicit, and simply utilize profits as the key factor determining investment without elaboration. Klein and Goldberger, for instance, merely state that "Investment, one of the dynamic forces in economic activity, is, by the assumptions of our model, motivated by profit".[62] Kaldor, in his "Model of the Trade Cycle" is about as brief, although he suggests the level of profits as the main influence on investment.[63]

In summary, profits are viewed by many as the major determinant of induced investment. The level of profits tends

59. Theory of Economic Dynamics, p. 98

60. Knox, p. 295

61. Ibid., p. 294

62. An Econometric Model of the United States 1929-1952 (Amsterdam: New Holland, 1955), p. 11.

63. Economic Journal, Vol. L (March, 1940), pp. 78-92.

to be more of a passive influence in that it provides funds for investment and facilitates obtaining of additional monies from external sources. The expected rate of profit on the other hand, provides the necessary impulse for entrepreneurs to undertake additional investment.[64]

As mentioned earlier, Tinbergen suggested the profits principle as an alternative to the acceleration concept. And as Somers states,

> Certainly, from some of the published work on the subject one might get the impression that the acceleration principle is something apart from the price or profit system.[65]

However, it is questionable as to whether these concepts should be thus separated. For example, the initial effect of an increase in the rate of output (and sales), will be, ceteris paribus, a rise in the rate of profit per unit of capital; thus profit expectations on additional investment will likely be improved, and the output of producer goods will be stimulated. In this instance, both the profits principle and acceleration principle are in evidence. The acceleration principle, whereby a rise in the rate of increase of output causes an enlargement of investment, has in fact operated through the change in profits.

Approached in this manner, it is evident that the

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64. Of interest is a study by T.C.Liu, and C.G.Chang, "Consumption and Income Propensities", American Economic Review, Vol. XL (September, 1950), pp. 565-582, in which the coefficient of multiple correlation of investment with income and the rate of corporate profits was as high as .97.

65. Somers, p. 86.

two concepts are closely related. The profits principle might therefore be considered as a broader explanation of induced investment activity. That is, investment may not only be stimulated by a change in output working through profits; other factors such as lower interest rates, lower prices of capital goods, improved quality of raw materials, more efficient labor, changes in technology, or better administration may also enhance profits and profit expectations and thus induce additional investment.

This discussion places the acceleration principle and the profits principle in perspective. More will be said on the relationship of these two concepts when discussing Matthews' approach to the business cycle in the succeeding chapter.

CHAPTER IV

THE ACCELERATOR IN BUSINESS CYCLES

The preceding chapter pointed up many of the qualifications of the acceleration principle. However, these well known limitations have not prevented the employment of this concept in the explanation of business cycles and more recently, dynamic growth models. The idea was also used for a time in the theory of pump priming. These applications must now be considered.

The Accelerator in Business Cycles: Early Approaches.

The application of the accelerator to business cycles is as old as the principle itself. T.N.Carver, in 1903, first submitted the idea, in a somewhat unpolished form, as an explanation of how business fluctuations might occur quite apart from the initiating forces of the money and financial markets.[1] He believed that a rise in consumer demand would raise the capitalized value of the plant and thus stimulate production of new capital equipment. Once the equipment was completed, new consumer goods would flood the market, reducing prices, profits and capitalized plant value, thus initiating a down turn, and so the process would go.[2]

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1. T.N.Carver, "A Suggestion for a Theory of Industrial Depressions", Quarterly Journal of Economics (May, 1903), pp. 497-500.

2. Ibid., pp. 499.

Aftalion, writing six years later, also believed that once a break from equilibrium occurred, periodic cycles would result through the operation of the acceleration principle.[3] He was persuaded that an initial upsurge in consumer demand would cause the following sequence: a shortage of consumer goods which would not be immediately satisfied because of a shortage of equipment--a rise in prices and profits--a need for a percentage increase in capital stock which equals the percentage increase in consumer demand, and which, because annual production of producer goods is only sufficient to cover replacement, results in a magnified percentage increase in the actual production of capital stock--a period of prosperity while this amplified demand for capital equipment is being satisfied--an overshooting of the mark on such production because the time-consuming nature of such production meant a continuation of high prices and profits even after the fabrication of sufficient equipment to meet the need had been undertaken--excessive production of consumption goods, low prices, depression--a gradual wearing out of fixed capital--a new scarcity of consumer goods, and the process would repeat itself.

Bickerdike in 1914 also discussed the acceleration idea in general terms using the ship building industry as an example of the type of fluctuations that would occur.[4]

3. The following treatment has been extracted from a discussion of Aftalion's views by A.H.Hansen, in Business Cycle Theory: Its Development and Present Status (Boston: Ginn, 1927) pp. 104-114.

4. C.F.Bickerdike, "A Non-Monetary Cause of Fluctuations in Employment", Economic Journal, Vol. XXIV (September, 1941), pp. 357-370.

These early endeavors to establish the accelerator as the mainspring of business cycles were naturally far from complete in their exposition of the principle itself or of its cyclical effects. J.M.Clark, in 1917, submitted a much more comprehensive discussion of the concept but did not attempt to develop it into a full explanation of the trade cycle.[5] Rather, he left it as a contributing factor. It was not until 1935, by which time the multiplier was becoming better known, that Clark hinted at the possibility of models based on the interaction of these two theories.[6]

Frisch used the accelerator in 1933 in his essay "Propagation Problems and Impulse Problems in Dynamic Economics.[7] It was his contention that erratic shocks gave the economic system the impulse that set it in motion. The accelerator formed part of the structural properties of the system which determined the cyclical cause of its movement once it was jolted from some initial situation. He did not introduce the multiplier concept.

Later Developments

The first formal statements of models utilizing

5. Clark, "Business Acceleration and...", pp. 217-235.

6. J.M.Clark, Strategic Factors in Business Cycles (New York: National Bureau of Economic Research, 1935, pp. 117-179; also see "Additional Notes on Business Acceleration and the Law of Demand", Preface to Social Economics, pp. 259-260.

7. In Economic Essays in Honour of Gustav Cassel (London: Allen and Unwin, 1933).

both the multiplier and accelerator emanated from Lundberg in 1935[8] and Harrod in 1936.[9] Lundberg used a mathematical approach. Harrod, using a literary approach, treated the accelerator as one of three 'dynamic determinants' of the trade cycle, the other two being the marginal propensity to save and the shift to profits.[10] But this period was also when Keynes' revolutionary General Theory was published, and Keynes' treatise excluded mention of the accelerator. Consequently, the synthesis of the multiplier and accelerator into a theory of cyclical fluctuations did not receive wide acceptance until after Paul Samuelson expressed it in concise form in 1939.[11] As his model has been the foundation for all subsequent variations of trade cycle theories based on this interaction process, it deserves some elaboration.

The Samuelson Model

Starting with national income at zero, Samuelson assumed a sustained government expenditure, G(t), of $1.00 took

8. Studies in the Theory of Economic Expansion, 2nd. ed. (New York: Kelley and Millman, 1955), esp. ch. 9.

9. The Trade Cycle: An Essay, esp. ch. 2.

10. Ibid.

11. "Interactions Between the Multiplier Analysis and the Principle of Acceleration," Review of Economic Statistics, Vol. XXI (May, 1939), pp. 75-78 and reprinted in Readings in Business Cycle Theory, pp. 261-269. Also see "A Synthesis of the Principle of Acceleration and the Multiplier", Journal of Political Economy, Vol. XLVII (December, 1939), pp. 786 - 797.

place. Consumption in each period was based on income of the previous period; i.e., $C(t) = \alpha Y(t-1)$ where $C(t)$ is consumption in period t, $Y(t-1)$ is income in period $(t-1)$, and α is the marginal propensity to consume.

Induced investment he assumed to be a function of consumption, expressing this relationship as $I(t) = B[C(t) - C(t-1)]$ where $I(t)$ is induced investment in period t, and B is the accelerator coefficient. Substituting for C, this may also be written as $I(t) = B\alpha Y(t-1) - B\alpha Y(t-2)$. In other words consumption was presumed to lag one period behind changed in income, whereas investment occurred in the same period as the alteration in consumption which induced it.[12]

Using various values for the accelerator and marginal propensity to consume he illustrated four different expansion patterns for national income. Then, starting with the basic equation $Y(t) = G(t) + C(t) + I(t)$ he substituted the above values for $G(t)$, $C(t)$ and $I(t)$ to arrive at a second order difference equation:[13]

$$Y(t) = 1 + \alpha Y(t-1) + B\alpha Y(t-1) - B\alpha Y(t-2), \text{ which}$$

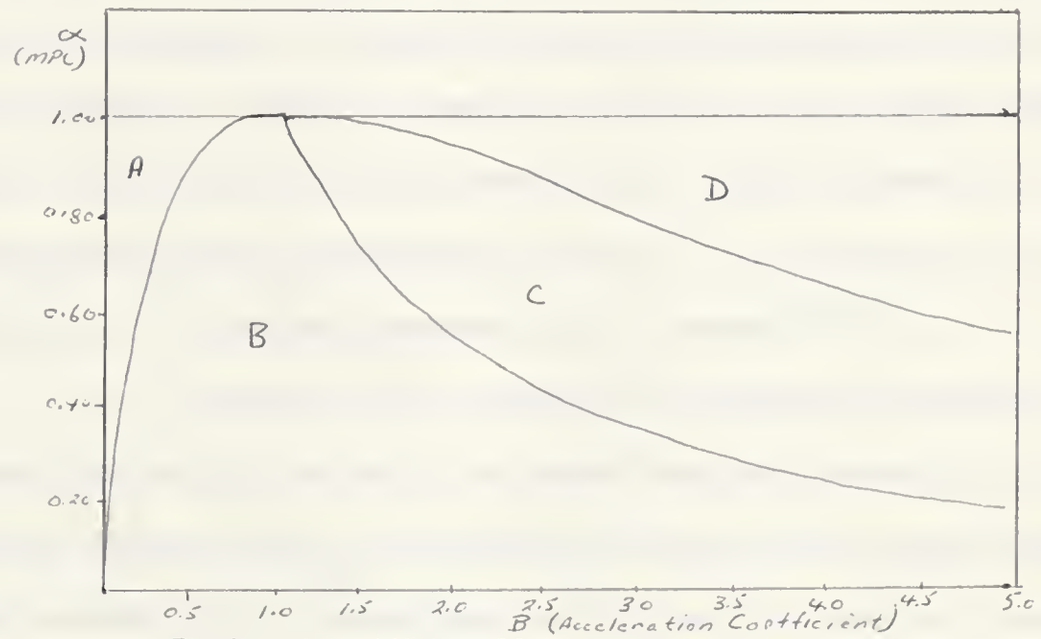
can be rewritten as $Y(t) = 1 + \alpha[1+B]Y(t-1) - \alpha BY(t-2)$.

12. These are the same lag assumptions used by D.H.Robertson in "Some Notes on Mr. Keynes' General Theory of Employment" Quarterly Journal of Economics, Vol. LI (May, 1936), pp. 168-191.

13. It might be noted that, as A. Smithies points out, Tinbergen, who was himself indebted to Frisch, provided economics with the linear difference equation as a tool for use in analysis. This set the pattern for nearly all the econometric work on business cycles that has attempted to link theory with the facts.

Thus, if the national income for two periods is know, the national income for the following period can be calculated. Without going into the details of the solution of this equation, he indicated that the whole range of possible values for the marginal propensity to consume and the accelerator could be divided into four regions, each of which would give a different behaviour pattern for income.[14] These regions are illustrated below.

FIGURE 8



Region A, with small values for the accelerator, resulted in income approaching asymptotically a value equal to what it would be with only the multiplier working, i.e., $\frac{1}{(1-MPS)}$ times the initial level of government spending. In region B, the values of the coefficient are such that national income

14. These details may be found in G.H.Fisher, "A Survey of the Theory of Induced Investment 1900-1940, Southern Economic Journal, Vol. XVIII (April, 1952) pp. 474-494

moves in a damped oscillatory manner, approaching the same final level as in Region A. In region C, ever enlarging oscillations around the level of income in Region A are the pattern. Along the line dividing regions B and C, the values of the coefficients would produce oscillations of constant magnitude around this level. Region D, with large values for the marginal propensity to consume and the accelerator coefficient, involves national income increasing continuously, i.e., explosive growth. And although in the other regions, if the additional government expenditure is discontinued, national income will return to its original level, this will not occur in this latter one; a single injection of net investment would be sufficient to drive "... the system up to infinity at a compound interest rate of growth."[15]

A review of one or two of the important features of this model will be valuable for comparison purposes with more recent constructions of this multiplier-accelerator relationship. The assumed lag in consumption results, as seen from the equation $Y(t)=1 + \alpha[1+B] Y(t-1)-\alpha BY(t-2)$. This "...direct dependence of Y(t) on Y(t-1) gives the system a tendency to cumulative movement up or down."[16] But this tendency is in turn

> ...restrained by the inverse dependence of Y(t) on Y(t-2). Cumulative movement away from the central equilibrium

15. Samuelson, "Interactions Between..." p. 268.

16. Matthews, p. 25.

> position is thus subject to a regulator, but the regulating mechanism works with a lag and is therefore liable to bring about fluctuations.[17]

Whether fluctuations actually occur depends, as has been indicated, on the values of the marginal propensity to consume and the accelerator.[18]

Samuelson's model also assumed that the magnitudes of the accelerator and propensity to consume were fixed in any given situation. This means that he did not allow for the accelerator to be reduced in the downswing to the size allowed by depreciation charges. Nor did it attempt to provide for the effects of the other qualifications which were shown in the previous chapter to alter this coefficient.[19]

A more elegant formulation of the multiplier-accelerator interaction process into a theory of the trade cycle was completed by J.R.Hicks in 1950.[20] In it a number of the limitations of the accelerator were either allowed for or at least discussed.

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17. Ibid.

18. Similar results would have been produced if the lag assumptions had been reversed, i.e., if investment had been lagged one period so that $I(t)=B[Y(t-1)-Y(t-2)]$ and consumption was unlagged so that $C(t)=\alpha Y(t)$. If neither consumption or investment had been lagged, the result would have been explosive growth. This is the situation assumed in the Harrod growth model discussed in the succeeding chapter.

19. Samuelson was well aware of the limitations of his model when he constructed it; pp. 269. His prime concern was to show the basic pattern that the multiplier-accelerator interaction produced without unnecessarily complicating the picture.

20. A Contribution...

The Hicksian Model

Hicks, like Samuelson, assumed that consumption was a function of past income; but rather than confining his analysis to the case were consumption depended solely on income of the previous period, i.e., $C(t) = \alpha Y(t-1)$, he also allowed for consumption to depend "... in part on income of the preceding period, in part on the income of the period before that... and so on for p past periods."[21] i.e., $C(t)=\alpha_1 Y(t-1) + \alpha_2 Y(t-2) +...+\alpha_p Y(t-p) + K$, where $\alpha_1 + \alpha_2 +... \alpha_p = \alpha$ or the marginal propensity to consume; K is a constant base level of consumption when income is zero.

Induced investment he made a lagged function of all output rather than an unlagged function of changes in consumption only, as in the Samuelson model: i.e., induced investment was assumed to be dependent on changes in output of the preceding period over the period prior to that. In his more complex case, investment depended upon output of a number of preceding periods i.e., $I(t)=v_1[Y(t-1) - Y(t-2)] + v_2 [Y(t-2) - Y(t-3)] +... + v_{p-1} [Y(t-p+1) - Y(t-p)]$, where $v_1 + v_2 +...+ v_{p-1} = v$, the accelerator coefficient.[22] Hicks interpreted the accelerator as applying only to the ratio of the initial change in output and subsequent induced investment in fixed capital;[23] the expansion

21. Ibid., p. 172

22. Ibid., p. 182, and ch. 4.

23. Hicks also discussed the possible changes that may occur in working capital (goods in process and minimum stocks of raw materials) and liquid capital (reserve stocks of materials not technically necessary to the production process, and finished goods), pp. 47-51, but these do not significantly alter the above statement relating to fixed capital. He also considered

of output which may occur once firms have enlarged capacity along their long run marginal cost curves, was not counted.[24]

Hicks also provided for autonomous investment, (investment which is independent of the level of output), which would rise at a fairly constant rate as determined by the growth of population and the rate of technological advance.

At any point in time there was assumed to be an upper limit on production beyond which output could not be increased. This was not to be regarded as a fixed level but rather one which was determined by the same factors responsible for the growth rate of autonomous investment. The two rates would thus be the same.

Then, starting with the system in equilibrium, with autonomous investment and output rising at constant rates, he assumed a single, extra burst of autonomous investment, and proceeded to demonstrate the behavioural patterns which would result. Essentially, these were the same as those discussed under the Samuelson model. In Hick's own words, "There are in principle four things which can happen; the four alternatives are readily distinguishable according to the values of the investment coefficient v."[25] These were: a level of output which

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23. replacement investment but concluded that its effects would not significantly change the analysis.

24. This point was discussed in chapter III of this paper, pp38. The important feature brought out in that discussion was that when such an assumption is used the acceleration coefficient will not be constant throughout the upswing as Hicks assumed.

25. By investment coefficient he means the acceleration coefficient.

increases for a few periods but soon returns to its equilibrium level, damped cycles, anti-damped cycles and explosive growth.

Hicks assumed that the accelerator would always be greater than one; in line with his other assumptions, this meant that either anti-damped cycles, or if uninhibited by the full employment ceiling, explosive growth, would be the result of the multiplier-accelerator interaction process. Take first the case of explosive growth.

In the diagram below, a semi-logarithmic scale is used to measure output (or investment) on the vertical axis. The line EE is the equilibrium path of output which depends on AA the course of autonomous investment. The line FF is the full employment ceiling determined by the same factors which determine autonomous investment; it is assumed to be above the

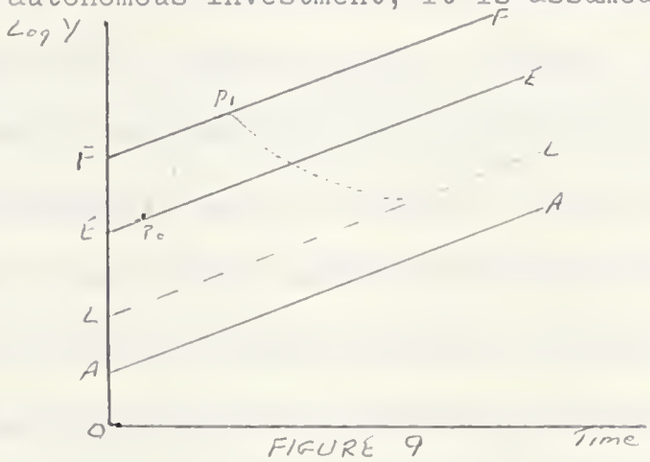

FIGURE 9

equilibrium path. It if is assumed that the economy is proceeding at an equilibrium pace and a "hump" of additional autonomous investment occurs at P_o, then output will commence rising faster than EE. This will continue until the ceiling is reached at P_1. Then the rate of growth of output will be slowed down at the ceiling rate. Thus, according to the acceleration effect, this slowing down will produce an absolute decline in induced

investment. Through the multiplier, output will soon follow the decline in induced investment, since autonomous investment will not be sufficient to sustain the former high growth rate.

Once output has started falling absolutely, the acceleration principle ceases to operate, or at least its effect is limited to the rate of natural depreciation. In other words, positive induced investment ceases, and disinvestment occurs up to the level allowed by depreciation. (Recognition of this fact was on improvement over the Samuelson model). Total investment in each period thus becomes equal to autonomous investment minus the disinvestment of previous induced investment. The level of output will drop to a new low equilibrium path "...determined by applying the multiplier...to the (adjusted) course of autonomous investment".[26] On the diagram of the preceding page, this new equilibrium path is seen to be LL. However, a new upswing will not commence in earnest until depreciation reduces idle capacity; and more important, until the steady increase in autonomous investment raises output sufficiently that excess capacity is once more utilized. Then, subsequent increases in output will again bring the accelerator into play, and the cumulative upswing will begin anew.

The introduction of a rising trend of autonomous investment appears to avoid the usual criticisms that may be launched at any cyclical theory that attempts to use the accelerator as the explanation of the upturn. These criticisms centre around the

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26. Hicks, p. 102.

fact that before the accelerator may operate all excess capacity must be eliminated. Matthews summarizes these objections:

> The a priori objection is that since negative net investment in fixed capital can take place only by the neglect of replacement, it is bound to be a very slow process. The difficulty of reducing the stock of capital within a short space of time...is attested by the excess capacity which is a well-known feature of the slump. If the recovery could not take place till substantial amounts of capital had worn out, slumps would be much more protracted affairs than we know them to be, and they would persistently be longer than booms (since there is not the same difficulty about raising the stock of capital as there is about reducing it)....The empirical objection is simply this: in most slumps negative net investment does not occur. Investment certainly falls in the slump, but it appears to be very exceptional for it to fall below the replacement level over the economy as a whole.[27]

But whether the recovery will be sufficiently less protracted when autonomous investment is interposed, to come within the bounds of realism, has still been questioned. For as Tsiang says,

> ...Hicks has obviously assumed tacitly that autonomous investment merely generates the effective demand for output but does not add to the output capacity of the economy.[28]

Tsiang goes on to point out that autonomous investment will likely increase productive capacity, and thus whether it will help to remove the general excess capacity of an economy will depend upon whether its effect in increasing demand through the multiplier more than offsets its capacity effect.[29]

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27. Matthews, pp. 161-162

28. Tsiang, p. 336-337.

29. Ibid., p. 337. Also note, the capacity effects of investment are discussed more thoroughly in the next chapter.

However, in a recession the marginal propensity to save will tend to be quite low and thus, ceteris paribus the multiplier effects would be considerably greater than one. Consequently the extra demand created by the autonomous investment will likely surpass considerably the extra capacity resulting, and therefore this investment will contribute to the upturn occurring sooner.

In the other case which Hicks believed might occur, viz., anti-damped cycles, the analysis is essentially the same as for explosive growth. However, the downturn is brought about by the inherent nature of the multiplier-accelerator interaction process, rather than by the full employment ceiling being reached.

Hick's model has been praised as presenting the acceleration principle in a far more elaborate and elegant form than had hitherto been attempted. It was also commended for its recognition of the non-linear accelerator. This accelerator differed between the upswing and the downswing because of the limitations imposed by depreciation in the downswing. However, it has also been severely critized, largely because it employs the acceleration principle in its rigid form as the sole explanation of induced investment.[30] Without going into a detailed

30. For example see S.S.Alexander, "Issues of Business Cycle theory raised by Mr. Hicks", American Economic Review, Vol. XVI (December, 1951), pp. 868-878; also, N. Kaldor, "Mr. Hicks on the Trade Cycle", Economic Journal Vol. LXI (December, 1951), pp. 833-847; also J.S.Duesenberry, "Hicks on the Trade Cycle" Quarterly Journal of Economics, Vol. LXIV (August, 1950)

discussion of these multifaricus criticisms, many of which, in any case, may be surmised from the qualifications discussed in the preceding chapter, an examination of one of the more recent approaches which utilizes a more general concept than the rigid accelerator should prove fruitful.

Matthews' Approach

Matthews recognizes that the acceleration principle cannot be considered the sole determinant of induced investment. Consequently, he supports the profits principle discussed in the closing pages of the foregoing chapter. To quote his own words:

> The central consideration affecting the inducement to do investment is profitability. Investment will be done if the expected profits represent an adequate return on the sum spent. The physical relation between output and capital is important only in so far as it influences the expected rate of return on investment.[31]

In other words, the rigid acceleration principle becomes only one factor affecting investment, which is determined in the final analysis by the rate of profits.

He assumes that when the accelerator is used as the unique determinant of investment, the acceleration coefficient (apart from the many qualifications which may affect it), is usually considered as synonomous with the normal capital-output ratio that involves "...neither strain nor under-utilisation

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30. pp. 464-476; as well as W.W.Rostow, "Some Notes on Mr. Hicks and History", American Economic Review, Vol. XLI (June, 1951), pp. 316-324.

31. Matthews, p. 34.

of capacity".[32] However, when the profits principle is endorsed, this latter ratio must be restated in terms of profitability. Thus the normal capital-output ratio becomes the "...capital-output ratio which causes the level of profit to be normal..."[33] And by normal profit is meant "...that level of profit which if expected for the future would lead to zero net investment."[34] In Marshallian long run equilibrium terms, normal profits are such that the marginal rate of return on capital is exactly equal to its supply price. Thus if such profits are expected for the future, there will be no inducement for firms to expand or contract, enter or leave the industry; net investment will therefore be zero. It is this type of situation of which Matthews is thinking. If profits were expected to be super-normal, there would be an incentive to invest; this would imply that the ratio of capital to output must be below normal. And conversely, if below normal profits were anticipated, this would imply that the capital - output ratio was above normal.

For the economy as a whole, one must think in terms of the ratio of all capital stock to national income. As Matthews states:

> The rate of profit and the inducement to invest will depend on the extent to which the ratio of capital to national income departs from the notional normal which would make the rate of return on capital at the

32. Ibid., p. 12

33. Ibid., p. 37n.

34. Ibid.,

> margin equal to its supply price. The actual size of the normal capital-output ratio in any given state of technical knowledge will depend on what the supply price of capital is, i.e., what is the minimum acceptable rate of return on capital.[35]

Once it is recognized that the normal capital-output ratio does not depend solely on a physical relationship of output to capital, i.e., that is not necessarily the ratio at which capital is neither under-utilized nor operated with considerable strain, then, Matthews argues, two of the important assumptions surrounding the formulation of the rigid accelerator will not hold strictly. More explicitly, take a standard formulation of the acceleration principle such as $I(t)=v[Y(t-1) - Y(t-2)]$. The two assumptions referred to are, that the stock of capital will, by the end of period t, be brought up to the level appropriate to the income of period t-1, and that secondly, the stock of capital at the beginning of period t will be precisely appropriate to the income of period t-2. If then, the normal capital-output ratio is interpreted as indicated above, there will no longer be anything inherent in the process of adjustment from period to period which will cause the capital stock to be brought up to the levels required by these assumptions.

> In so far as there is a tendency for the stock of capital to be adjusted towards this level, it is as a result of the working of competition, not because anyone is consciously aiming at it. There is no question of firms planning to bring about long-period equilibrium, whether in one period or several. There is therefore no presumption that the amount of investment carried out during a period--defined in the sense of a planning period--will be exactly equal to the difference between the opening stock of capital

35. Ibid., p. 35.

> and that indicated as appropriate by the then prevailing level of income.[36]

Firms' reactions to a departure of the rate of profit from normal may be sluggish so that investment in any period may fall short of the required amount. Similarly, if reactions are very prompt, there may be an overshooting of the required mark because firms may not take into account the additional investment that others may also be undertaking to meet the current situation. This will hold true particularly where the gestation period in long, because then there may be a considerable time elapse before any investment already undertaken is placed into operation.[37]

To remedy these weaknesses of the acceleration principle which render it, when combined with the multiplier, an inadequate explanation of cyclical fluctuations, Matthews submits the capital stock adjustment principle which states that investment decisions will vary directly with the level of national income and inversely with stock of capital in existence. This may be expressed as $I(t)=aY(t-1) - bK(t)$, where, as usual, $I(t)$ is investment in period t, $Y(t-1)$ is income in period $t-1$, and $K(t)$ is the stock of capital at the beginning of period t;

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36. Ibid., p. 38

37. It is worth noting that these factors which may prevent the stock of capital from being adjusted exactly to the level of output as required by the accelerator, may apply regardless of whether one interprets the capital-output ratio in terms of Marshallian long-run normal profits, or as a simple physical ratio. However, the point of importance here is that the accelerator concept, in its rigid form has these deficiencies. It is Matthews' method of allowing for these factors which must now be examined.

of period t; a and b are constants. As Matthew states;

> This incorporates the basic idea of the acceleration principle, that investment will be directed towards bringing the stock of capital into alignment with the level of income recently prevailing, without attempting any undue precision.[38]

By considering the effects of the stock of capital, this model makes allowance for the possibility of excess capacity, one of the factors which may limit the operation of the rigid acceleration principle. Furthermore, by recognizing the effects of changes in national income in relation to capital stock, this formulation also implies the profits principle--that investment will be determined by the rate of profit per unit of capital.

The acceleration concept may be derived from this equation, when "...a is equal to the normal capital-output ratio and b is equal to unity."[39] This may be illustrated as follows:

If a=v=the normal capital output ratio, and b=1, then the capital stock adjustment principle may be illustrated as $I(t)=vY(t-1) - K(t)$. This may now be interpreted as saying that investment in period t, will be determined by the difference between the desired and the actual stock of capital at the beginning of the period; K(t) is the capital stock at the beginning of the period, and $vY(t-1)$ is the desired stock at the end of the period which in turn depends on income of the previous period t-1. Similarly, $I(t-1)=vY(t-2) - K(t-1)$

38. Ibid., p. 41

39. Ibid.

Rearranging this, one may arrive at

$$vY(t-2)=K(t-1) + I(t-1)$$

or $$vY(t-2)=K(t)$$

Then substituting for K(t) in the first equation above, one arrives at

$$I(t)=vY(t-1) - vY(t-2)$$

or $I(t)=v[Y(t-1) - Y(t-2)]$ which is a familiar form of the acceleration principle.

Matthews also discusses expectations and points out how these may affect the capital stock adjustment principle to some extent. But he does not provide for them explicitly in his investment equation.

The capital stock adjustment principle may be combined with the multiplier to produce cyclical cyclical models. If an investment lag of one period is assumed, as in the equation above, then the interaction process will produce the same four possibilities described in the Samuelson and Hicks models.[40] These are, convergence to equilibrium, explosive growth, dampened cycles, and anti-damped cycles. But in these patterns it is the rate of profits which stimulates investment rather than simply the rate of change of output as in the Samuelson-Hicks models.

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40. He also mentions that where no lags are assumed, and thus investment is a function of current income and the stock of capital, i.e., $I(t)=aY(t) - bK(t)$, there may not be complete instability as the rigid acceleration principle would produce, (and as will be discussed in the following chapter), but instead stable equilibrium may also result; pp. 50-51.

The importance of this model in the present context, is that it exemplifies how the acceleration principle in a generalized form has been used in recent explanations of business cycles.

On the empirical side, the capital stock adjustment principle has not been subject to as much testing as the rigid acceleration principle. One example however is the work done by Chenery, one of the first to expound this concept.[41]

He tested it, as well as the accelerator, in a number of industries over a 17 year period. As a whole, the former produced the more satisfactory results. It was found to be most superior to the acceleration principle in those industries where some overcapacity existed; this was as expected since its formulation expressly provides for the influence of the existing stock of capital.

Other Points of View

A number of business cycle theorists have rejected the acceleration principle and consequently, also the explanations of the cycle which are based on the accelerator-multiplier interaction. Tinbergen, Klien, Kalecki and Kaldor are usually classed in this category.

However, as one or two recent writers have pointed out, even the cyclical models of these men contain, in one form

41. Chenery, "Overcapacity..." R.M.Goodwin also developed a model of this type about the same time as Chenery; Econometrics in Business Cycle Analysis", in A.H.Hansen, Business Cycles and National Income, pp. 417-468

or another, the essential relationship of the acceleration principle. Matthews for example, mentions that in L.R.Klein and A.S.Goldberger, An Econometric Model of the United States 1929-1952[42] there is an investment equation similar in essentials to the capital stock adjustment principle as formulated above except that lags are not provided for. He adds that when one introduces buffers (floors or ceilings) into the analysis, one arrives at a cycle theory which corresponds to the cyclical models of N.Kaldor in, "A Model of the Trade Cycle",[43] and further, when one makes provision for lags, the resultant cycle theory corresponds to M. Kalecki's model.[44]

Eckaus also has taken the view that the models of these men contain the underlying concept of the accelerator principle--that investment depends upon output.[45] As mentioned in the previous chapter all of them recognize the rate of output or profits and the level of capital stock as the important variables in the investment relation.

To illustrate, one of Klein's models is as follows:

$$I=\beta_0 +\beta_1 \frac{pX-E}{q} +\beta_2 \left(\frac{pX-E}{q}\right)_{-1} +\beta_3 K_{-1} +\beta_4 t + u_2$$

where I is net investment in constant dollars, pX is the value of privately produced output, E stands for excise tax payments,

42. (Amsterdam: North Holland, 1955)

43. Economic Journal, Vol. L (March, 1940), pp. 78-92

44. Kalecki's most recent model is in his Theory of Economic Dynamics Part 5.

45. Eckaus, pp. 223-230.

K is the stock of capital at the end of the previous year in constant dollars, q is the price of producer goods, t is time introduced as a specific allowance for trend, the α's are constants, and u is a random disturbance.

One of Kalecki's models, arranged as an equation for the level of investment is:

$$I_{t+\epsilon} = \frac{a}{2\epsilon} P_t - \frac{(b+c)}{2\epsilon} K_t + \text{a constant},$$

where I is the rate of investment, P is profits and t is time. The letters a, b and c stand for parameters reflecting respectively the influence on investment of changes in the price of capital goods, the repressive influence on new investment of the existing capital stock, and the effects of risk, market imperfections and the degree of indebtedness of entrepreneurs; ϵ is the total lag of a change in the rate of investment behind changes in the level of profits.

With the use of a diagram as that shown opposite, it is possible to show that the accelerator may also be formulated in a manner similar to the above investment equations. This diagram must be viewed as an isoquant map turned on its side so that the level of output becomes visible. It assumes that there are only two

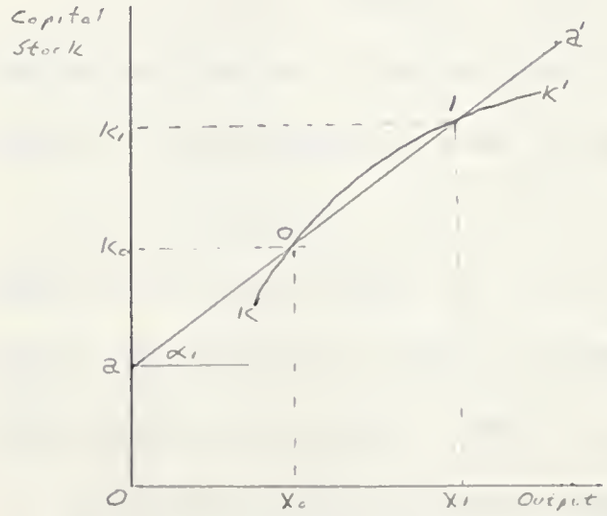

FIGURE 10

factors of production, labor and capital, and that to increase output, both factors are increased in accord with the usual "least-cost combination" method common to theory of the firm analysis as discussed in Chapter II of this paper. The curve KK' indicates the aggregated capital equipment which businessmen require for each level of output. Price changes, uncertainty, indivisibilities, and other factors affecting investment decisions may all be assumed to be reflected in the curve, which in this instance shows increasing returns to scale; i.e., the accelerator is declining as additional units of capital are employed.

It is seen that K_o is the level of stock required to produce X_o. If X_1 units of output are to be produced, K_1 units of capital stock will be required. If the line aa' is drawn through the points O and 1, it makes the angle α_1 with a perpendicular to the Y-axis at point a. The acceleration principle can be formulated in terms of this diagram as follows:

$I_1 = K_1 - K_o = (\tan\alpha_1)(X_1 - X_o)$. In this instance, $\tan\alpha_1$ is the accelerator. Reading from the diagram again, an alternative method of expressing the same thing is:

$I_1 = (\tan\alpha_1) X_1 - K_o + a$. This equation can be seen to express basically the same thing as do the Klein and Kalecki models shown above, (although the latter also make provision for a number of other variables as well.) This is understandable, since as Eckaus says:

> From each point of view the firm compares the future output which it wants to produce with its existing productive capacity, and invests if added productive capacity is needed to make up the difference.[46]

46. Ibid., p. 227.

The only real difference is that, as with the capital stock adjustment principle, the Klein and Kalecki type of equations make express allowance for the influence of the initial amount of capital equipment, whereas the normal formulations of the accelerator do not.

Summary

The discussions of this chapter have included a brief outline of the history of the application of the acceleration principle to business cycles. One or two models of the business cycle which utilized the accelerator in its rigid form (along with the multiplier) were then reviewed. It was seen that, depending upon the size of the propensity to save, and more important, the magnitude of the accelerator coefficient, four possible growth patterns may result when consumption, investment, or both, operate with a lag. Then, a cyclical model based on profits principle which recognizes that other factors besides the accelerator may affect induced investment, was explained. It was shown that from the investment equation involved (referred to as the capital stock adjustment principle) the rigid accelerator principle could, under special circumstances, be derived.

Note was also made of the views of some of those who reject the acceleration principle. But it was shown that even the models of these men could easily be interpreted as containing this principle, although in a generalized form.

It might be noted in concluding this section that no mention has been made of a number of the writers whose works

certainly deserve attention in any discussion of the accelerator in business cycles analysis. Alexander, Baumol, Bennion, and Metzler, to list only a few, would fall into this category.[47] But the omision has been intentional. The prime purposes here have been first to indicate the manner in which the accelerator has been used to explain trade cycles, and secondly, to illustrate the two opposing streams of thought as to the value of this principle and the manner in which these streams may be reconciled.

The Acceleration Principle and Pump Priming

The pump priming concept is well described by Hansen in Fiscal Policy and Business Cycles. It involves both the multiplier and accelerator principles and

47. S.S. Alexander, "The Accelerator as a Generator of Steady Growth", Quarterly Journal of Economics, Vol. LXIII (May, 1949), pp. 174-197.

W.J. Baumol, "Notes on Some Dynamic Models," Economic Journal, Vol. LVIII (December, 1948), pp. 506-521.

E.G. Bennion, "The Multiplier, the Acceleration Principle and Fluctuating Autonomous Investment", Review of Economic Statistics, Vol. XXXVII (May, 1945), pp. 85-92.

L. A. Metzler, "The Nature and Stability of Inventory Cycles", Review of Economics and Statistics, Vol. XXIII (August, 1941), pp. 113-129; and "Factors Governing the Length of Inventory Cycles", Review of Economics and Statistics, Vol. XXIX (February, 1947), pp. 1-5; also, "Business Cycles and the Modern Theory of Employment," American Economic Review, Vol. XXXVI (June, 1946), pp. 278-291. Metzler showed that the acceleration principle was particularly suitable to the explanation of inventory cycles. And it is noteworthy, that even some of the opponents of this principle recognize that it does seem applicable to this area. eg. Kalecki, Theory of Economic Dynamics, p. 106.

> ...carries with it the implication that a certain volume of public spending, varying under different conditions, will have the effect of setting the economy going on the way toward full utilization of resources on its own power without further aid from governmental spending.[48]

It differs from the idea of public compensatory spending in that this latter concept does not imply anything about setting the system going on its own mementum. 'Compensation' strictly interpreted implies only that public expenditures may be used to compensate for the decline in private investment. The difference between the two terms is thus primarily a matter of intent as to what it is sought to achieve.

Hansen uses Samuelson's interaction model to discuss the effectiveness of pump priming methods. He points out, as did Samuelson, that only when the values for the marginal propensity to consume and the multiplier are large is it possible that a single injection of government investment could stimulate the economy to a sustained recovery. However he felt that such values for these coefficients would rarely be achieved, and concluded that the theory of pump-priming based on the accelerator-multiplier effect was not of great practical value for economics in depression stages.

Those who believe that the accelerator is large enough (when accompanied by the multiplier) to cause exponential growth in income (such as Hicks) could argue in favor of pump priming. However, there has been little mention of

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48. p. 262

this concept in recent economic literature, interest having turned more to the problems of continuous long run growth.

CHAPTER V

THE ACCELERATOR IN THE THEORY OF ECONOMIC GROWTH

The previous chapter, which attempted a summary outline of the use of the acceleration principle in business cycle analysis made the implicit assumption that it was static rather than dynamic analysis that was involved. No consideration[1] was given to the fact that investment, besides generating income through the operation of the multiplier, also increases productive capacity and thus requires a growing level of aggregate expenditures in order to absorb the additional output of this enlarged capacity.

Nor was there consideration of the fact that with growing income, the absolute volume of savings tends to increase so that investment must be continuously increased to absorb these larger savings. Dynamic analysis, which as Baumol describes, is "...the study of economic phenomena in relation to preceding and succeeding events,"[2] considers these problems.

The concern in this chapter is to examine the developments in thinking along these lines to determine the place the accelerator holds in dynamic growth theory. First, a brief review will be made of the growth models of Harrod and Domar who, as is well known, did the pioneer work in this field.[3]

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1. With the exception of Tsiang's comments on Hicks' analysis p. 77.

2. W.J. Baumol, Economics Dynamics: An Introduction (New York: Macmillan, 2nd. ed. 1959) p. 4.

3. It is worth noting that Professor Ivo Moravcik pointed out in his Doctoral thesis, The Great Desideratum:

Since their initial work, the literature may be divided into two main groups -- that dealing with whether the Harrod-Domar models are stable or unstable and thus whether they can be considered realistic representations of how growth may occur; and secondly, that concerned with the formulation of other long run growth models. Two approaches to this secondproblem can be distinguished: that based on the theory of distribution, such as the work of Mrs. Joan Robinson and Mr. Kaldor, which looks at the interdependence of the rate of profit and the pace of capital accumulation as well as the effects of the distribution of income on the proportion of income saved; and that based on the theory of the trade cycle.

The nature of the discussions on the stability of the Harrod-Domar models (which involve the use of the accelerator concept) will be examined. No further mention will be made of the problem of growth based on distribution theories, as these do not involve the accelerator. As for those theories based on trade cycle analysis, some involve the use of the accelerator and others do not. An attempt will be made to outline the opinions of several of the major writers in the field, with particular reference to the importance placed on the acceleration principle.

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3. The Soviet Search for Industrial Expansion (Bloomington: Indiana University, 1959), pp. 161-164, that as early as 1928 a Russian economist, Fel'dman, divided the economy into two sectors. For one of these sectors he developed a growth rate which is equivalent to the averagepropensity to save times the productivity of capital. As Moravcik states, "This resembles closely the warranted rate of growth of Harrod and the equilibrium rate of growth of Domar." p. 163.

Domar's Model

Domar's model assumes the following: (a) a constant general price level, (b) no lags, (c) savings and investment refer to income of the same period and are both net of depreciation, (d) no government and no foreign trade, (e) a state of full employment exists,(f) the average and marginal propensities to save are equal and constant, and (g) the capital coefficient is constant.[4]

On the supply side: $\Delta Y = I\sigma$ i.e., the total increase in productive capacity, ΔY, in any period (say one year) equals the net investment, I, multiplied by, σ, the inverse of the capital coefficient or the productivity of capital[5] (net of all complications).

On the demand side: $\Delta Y = \Delta I(\frac{1}{\alpha})$, i.e., the total increase in income, ΔY, in a given period will equal the change in investment since the previous period, ΔI, times one over the marginal propensity to save, α, i.e., times the multiplier, $\frac{1}{\alpha}$. The change in income is a function not of the level of net investment (as the change in productive capacity is) but of the increment in investment, ΔI.

4. "Capital Expansion, Rate of Growth and Employment", Econometrica, Vol. XIV (April, 1946), pp. 137-147, and reprinted in Essays in the Theory of Economic Growth (New York: Oxford, 1957), pp. 70-82, also "Expansion and Employment", American Economic Review, Vol. XXXVII (March, 1947), pp. 34-55, and reprinted in Essays.. pp. 83-108.

5. The 'productivity of capital' as used by Domar allows for the increase of the other factors of production in combination with the increase in capital. Thus it must not be construed as meaning either the average or marginal product of capital. These terms, as mentioned in Chapter II, refer to the increase in output when increments of capital only are added to the production process while the amounts of other factors are held constant.

To maintain full employment equilibrium, the increase in productive capacity must equal the rise in expenditures on the demand side; i.e., $I\sigma = \Delta I(\frac{1}{\alpha})$. When rearranged this gives $\frac{\Delta I}{I} = \sigma\alpha$. This means that investment must grow at the rate $\sigma\alpha$. And since the marginal and average propensities to save are assumed to be constant, income thus must grow at the same constant compound rate as is indicated for investment. This means too that the absolute increases in investment and income in each period must grow continuously. If income does not grow at this rate, then either the new capital will remain unused, the new capital is used at the expense of the previously constructed capital, or the new capital may be substituted for labor and possibly other factors.[6]

Harrod's Model[7]

Harrod bases his model on the assumptions (a) that the net saving of the community, S, in a period, t, is a constant porportion, s, of the income received during the period, Y(t); (b) that actual saving equals intended saving (thus he is assuming that actual and expected income are equal); and since actual savings always equal actual investment, actual investment

6. Income could only remain at a constant level and full employment of capital be maintained if and only if net investment and net savings both equal zero. Gross investment would then be sufficient to cover replacement only.

7. This discussion is, unless otherwise stated, from R.F. Harrod, "Essay in Dynamic Theory", Economic Journal, Vol. XLIX (March, 1939), pp. 14-33; and Towards a Dynamic Economics, (London: Macmillan, 1948).

will equal actual and intended savings; and (c) that intended investment, I, in period t is determined by the rate of increase of income; or in greater detail, the desired investment, I(t), of any period t will be a constant proportion, Cr, of the difference in income in that period, Y(t), and the income of the preceding period, Y(t-1). This is then, the acceleration principle in operation. Thus from the first and second assumptions one gets $S(t)=sY(t)$. And from the third assumption, one obtains $I(t) = Cr[Y(t) - Y(t-1)]$. Investment desires will only be realized if and only if $sY(t)=Cr[Y(t) - Y(t-1)]$ which if rearranged will give $\frac{Y(t) - Y(t-1)}{Y(t)} = \frac{s}{Cr}$ or $Gw = \frac{s}{Cr}$ where Gw is the warranted rate of growth, or the rate whichif achieved will leave entrepreneurs, satisfied with the investment they are currently doing and prepared to carry on a similar advance.

Similarity of the Two Models

Although some have tried to argue that these two models are actually quite different,[8] they are, as Harrod reaffirms,[9] basically the same. Harrod's s, or marginal propensity to save,

8. e.g., H.T. Oshima, "Income Originating in the Models of Harrod and Domer", Economic Journal, Vol. LXIX (September, 1959), pp. 443-450.

9. "Domar and Dynamic Economics", Economic Journal, Vol. LXIX (September, 1959), pp. 451-464; It might be noted that although Harrod does argue that the two models are basically the same, he also suggests that possibly Domar's growth rate $\alpha\sigma$, might be interpreted as equivalent to his ownnatural rate of growth which is defined on p. 100 of this paper. Ibid., p. 456; also see Domar, "Expansion and Employment", p. 95-96.

is the same as Domar's α; Harrod's Cr is the number of units of new investment, properly utilized, that are necessary to produce an extra unit of output; thus $\frac{1}{Cr}$ equals Domar's α, since the latter is also valued on the basis that new investment is properly utilized; and since under Domar's assumptions $\frac{\Delta Y}{Y} = \frac{\Delta I}{I}$, the growth rate, $\frac{\Delta Y}{Y}$, so determed is equal to Harrod's Gw. Although Domar's approach refers only to the capacity effect of new investment, it can be easily interpreted as recognizing the acceleration principle.

It is on these two models, involving the propensity to save and the accelerator or its inverse, that a large number of all subsequent theories of growth have been based.

The Capacity Principle and the Acceleration Principle

In working with growth models, some writers refer to the capacity principle rather than the acceleration principle. Similarly they use the term marginal capital coefficient rather than the acceleration coefficient.[10] Essentially, there is no difference between these terms insofar as they are applied to the above growth models as developed up to this point.

Primarily, only a change of emphasis is involved. The acceleration principle, in its generally accepted form, imports a change of output which in turn induces an alteration in investment; the magnitude of the change in investment is

10. e.g., A.E. Ott, "Relation Between the Accelerator and the Capital-Output Ratio", Review of Economic Studies, Vol. XXV (June, 1958), pp. 190-196.

determined by the acceleration coefficient (all qualifications assumed aside). The capacity principle implies that the change in investment occurs first and thus enlarges capacity (assuming it was an increase of investment) so that an expansion of output may follow; the ratio of the change in capital to the possible increase of output, is termed the marginal capital coefficient. In both instances, if technology and all other influences are held constant, the accelerator and the marginal capital coefficient will be identical. In other words, these concepts differ only as to the direction of causation implied. And if it is remembered that in the growth models under consideration, it is assumed that there are no period lags between the changes in output and the related changes in investment, the question of which one precedes the other is of little consequence.

The only situation where a distinction between these terms becomes meaningful is when non-induced or autonomous investment is introduced. In this instance, the marginal capital coefficient or ratio of the increment of autonomous induced investment to total increment of output, may well differ from the accelerator, which refers only to the ratio of the change of induced investment to that portion of output which is relevant. Then, only if the productivity of the autonomous investment is the same as that of the induced, will these terms be equal. However, unless autonomous investment is specifically included in the discussions which follow, this distinction need not be of concern; the capital coefficient and accelerator may be treated as synonomous.

Effect of Changes in the Accelerator

The crucial elements in the Harrod-Domar models are the propensity to save and the accelerator or capital coefficient. Changes in either of these will alter the rate of growth required for full utilization of capital. It is the capital coefficient which is of prime concern here. The effects of alterations in its magnitude, when the propensity to save is held constant, are illustrated in the following table:

TABLE 4

CHANGES IN THE CAPITAL COEFFICIENT

Propensity to Save (s)	Capital Coefficient (Cr)	"Productivity" of Capital (1/Cr)	Warranted Growth Rate (Gw)
.2	5	.20	.04
.2	4	.25	.05
.2	3	.33	.06

(where $Gw = \frac{s}{Cr}$)

The larger the capital coefficient the lower the rate of growth of income necessary for new capital to be utilized fully, i.e., the more units of capital required to produce an additional unit of output, the lower the rate of growth required to utilize additional units of capital.

The Razor's Edge:

In addition to his warranted rate of growth, which if achieved will cause investors' plans to be realized, Harrod defined a natural rate of growth, Gn, which is the maximum rate of growth possible, given the rate of growth of the labor force, natural resources, and technology. If Gn is the same as

Gw, then it is conceivable that the economy could proceed at this rate. Should a change take place in the key determinants of Gw, namely the saving ratio or the accelerator; or more important, should the rate of growth of the labor force or technology, the main determinants of Gn, alter in either direction, then ceteris paribus, the actual rate of growth, G, could depart from Gw and cumulative divergence of the two rates would ensue.[11] That is, the long run rate of growth of the economy represented by Gw is unstable. The economic system is balanced on a razor's edge of equilibrium growth. This is the often quoted characteristic of the non-lagged Harrod-Domar model.

The rationale is as follows: Should G rise above Gw, it would mean, if one looks at it from a period analysis point of view, that there was a greater call for capital goods than entrepreneurs had anticipated in that period. Thus, stocks would be depleted and equipment over-worked. In the following period entrepreneurs will enlarge investment expenditures at an even faster rate, which in turn, through the multiplier effect of this investment will expand income (and demand) so that there is an even greater deficiency of investment. And so the process will continue. Attempts to catch up by businessmen only widen the gap.

Similarly if some disturbance causes G to fall below

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11. Conceivably, the actual rate, G, may remain at Gw for a time, but if conditions remained out of balance, eventually cumulative divergance would occur, e.g., more intensively at first, but if this pace could not be sustained, then G would drop below Gw.

Gw, investors' expectations will be disappointed; in the following period they will invest at a slower rate, and thus the actual rate of growth will decline even further below the warranted rate. Departures of the actual rate either above or below the warranted rate will lead to cumulative discrepancies between the two rates.

Harrod's Limitations on Instability

Harrod examined the possibility of this instability being prevented by changes in the values of s and Cr as a result of changes in the actual growth rate. He concluded that changes in Cr, the accelerator, would augment the instability rather than remove it. For example, in a depression situation, where output is increasing at a rate less than is warranted for investors' expectations to be realized, then it is likely that there will be excess capacity so that a given rise in output will induce little if any net investment; i.e., the accelerator will be very small if it exists at all, and Gw will thus be pushed further above G. Conversely, in prosperous times, with G much above Gw, there will likely be little excess capacity so that Cr will be large, and Gw therefore will be depressed even further below G.

On the other hand, changes in savings will tend to move so as to reduce the discrepancy between the growth rates. For example, if G is above Gw, and as a result, the actual level of income rises substantially, people may well tend to save a larger proportion of their incomes. This would increase Gw so as to bring it closer to the actual rate. This may be seen from

the equation $Gw = \frac{s}{Cr}$. More specifically, it means that with higher savings, realized investment will also be increased, and as Baumol states;

> This added investment will take care of some of the investment demand that would not otherwise have been satisfied. The rise in s will thus help offset the underproduction resulting from the rise in output above the warranted output.[12]

The opposite type of reaction would tend to occur when G falls below Gw.

But in either situation, it is extremely doubtful that such changes would ever be sufficient to prevent the model from being unstable. This may be illustrated quite easily. If G rises above Gw, then, let x be the increase in income in a single period t, s_m the proportion of this income saved, (i.e., the marginal propensity to save), C.x the excess investment demand resulting from this additional income, C the ratio of the extra investment that will be demanded during period t, over the rise in output during the same period above that of period t-1 (or the capital coefficient). And if C.x is to be balanced by saving, then $s_m.x = C.x$, i.e., $s_m = C$. This means that in period t, the marginal propensity to save must be equal to the capital coefficient. Since the capital coefficient will usually be above 1,[13] the propensity to save would have to be likewise; thus the increase in savings must exceed the

12. Baumol, Economic Dynamics, p. 51.

13. See D. Hamberg, Economic Growth and Instability: A Study on the Problem of Capital Accumulation, Employment, and the Business Cycle. (New York, Norton 1956), pp. 36, 45n.

increment of income. Because this appears to be an extremely unlikely occurrence, Harrod concluded that the adjustment in savings would not be sufficient to bring Gw again to equality with the actual growth rate, once the two have separated.

Introduction of Autonomous Investment

Harrod also introduced other than induced investment into his model. This included investment determined by the level of income rather than its rate of change, as well as that which depends neither on the level or rate of change of income such as innovational capital. If the former is expressed as a fraction of income (designated by k) and the latter as an absolute amount (K), it may be shown[14] that the formulation for Gw becomes $Gw = \frac{s-k-\frac{K}{Yt}}{Cr}$.

The effect of this type of investment on Gw may be clearly seen if one starts from a position where $Gw = \frac{s}{Cr}$ and then introduces k and $\frac{K}{Yt}$. The result is that the numerator is reduced and thus Gw is decreased; i.e., the growth rate of income now required in order to induce, through the acceleration principle, sufficient intended investment to offset intended savings in less than originally, because a portion of such savings are now balanced off by the non-induced type of investment.[15]

14. Harrod, "Essay...", p.27.

15. This would be a situation where, if the productivity of the non-induced investment (when other factors are being increased at the same time) differed from that of the induced investment, the marginal capital coefficient relating to total investment would differ from the acceleration coefficient.

On the other hand should other types of investment be discontinued or even fail to increase at a rate as high as the rise in income, then the numerator and Gw would rise; or, in economic terms, the rate of increase of income must rise much more in order to induce sufficient investment to offset the savings which are no longer offset by this "autonomous" investment. Thus, although the acceleration principle is no longer the sole factor determining investment, it remains an important part of the model.

Considerable space has been devoted to this basic type of growth model, and the place of the accelerator principle in it. This has been necessary to set the stage for a look at the more recent thinking regarding such models and the importance of the accelerator in them.

Stable of Unstable Pattern of Growth

Numerous arguments have been submitted to prove that the Harrod-Domar Model is not unstable.[16] For example, Yeager suggested that the instability of the system could be held in

16. It may of course be argued that by introducing lags, such as in the Samuelson or Hicks models, Harrod's model may then take a form similar to these earlier models, where four patterns of behaviour of income are realistic possibilities; if this were so, instability, or a cumulative departure of G from Gw need not then necessarily occur. But the lag assumptions of the Samuelson and Hicks models are also open to question. Thus the approach has been to try to prove that the Harrod model may still be stable even when Harrod's assumptions of no lags are accepted.

restraint by the same measures which are used to prevent monetary inflation and deflation, i.e., by holding a tight rein on the money supply when G is greater than Gw, and increasing the money supply when G is less than Gw.[17] Fellner also submitted that the movement from the equilibrium rate would tend to be reversed over time due to monetary policies, and also felt that the variations in saving and the capital output ratio first mentioned by Harrod would be sufficient to bring stability if producers chose to make them complete.[18]

Baumol pointed that the general state of producers' expectations may also affect investment so as to check the cumulative movement of G away from Gw once some disturbance occurs. He makes this clear in relation to a situation where overproduction (or excess supply of goods) occurs, i.e., $G<Gw$.

> ...if income has been rising at the warranted rate for some time before the difficulty in question arises, entrepreneurs may refuse to be moved to pessimism by a single unhappy experience. It is even possible that entrepreneurs will generally believe in the "normalcy" of the economy and so will regard any case of over production as a temporary phenomenon soon likely to disappear.
>
> If any such possibility applies, it no longer necessarily follows that a situation involving overproduction must result in such a contraction of output plans that the warranted rate of growth will fail to be reattained.[19]

Similar reasoning may be applied to the case where $G>Gw$.

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17. "Some Questions about Growth Economics", American Economic Review, Vol. XLIV (March, 1954), pp. 53-63.

18. "The Capital-Output Ratio in Dynamic Economics" in Money, Trade, and Economic Growth in Honor of John H. Williams (New York: Macmillan, 1951).

19. Baumol, Economic Dynamics, pp. 54-55.

Furthermore, once G departs from Gw, then Gw itself shifts in such a manner as to bring the two rates closer together again. For example, if G declines below Gw, and thus investment is reduced, it will mean that the rate of growth now required for full utilization of capital will itself be lessened. In the diagram below, if the actual income growth falls short of Y" at time t, then with investment being decreased the required rate of growth is also reduced so that the balance of the YY' curve to the right of Y" is no longer relevant. The new warranted rate will be as represented by $Y''_1Y'_1$. The same type of analysis may be used where G > Gw.

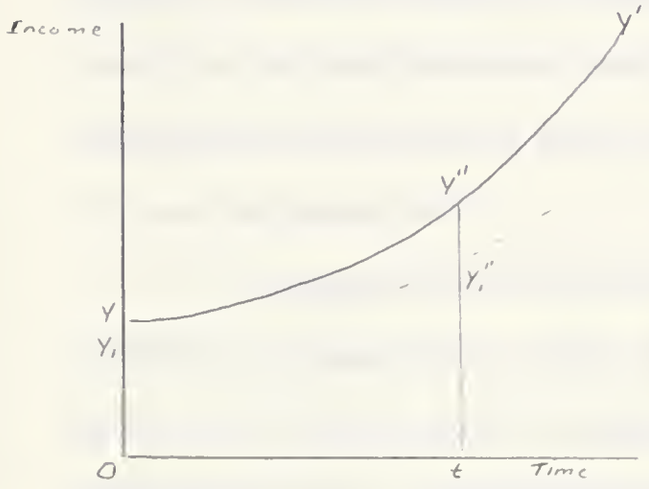

FIGURE 11

The big break-through in establishing the potential stability of the Harrod-Domar model came with Solow's article in 1956, "Contribution to the Theory of Economic Growth".[20] His approach and conclusion are what are of prime importance here. He showed that by relaxing the assumption that production occurs under conditions of fixed proportions, and allowing for an increase in the labor force, that the razor's edge situation need not occur. In his conclusions he also provided for neutral technological change and an interest elastic

20. Quarterly Journal Economics, Vol. LXX (February, 1956), pp. 65-94.

savings schedule.

A number of papers have since been published which, starting from Solow's analysis, introduce other variables and assumptions. For example, Bruton assumes that the rate of growth of population is zero; then he suggests that innovations will be introduced to prevent the growth of per capita income from falling. He also argues that such innovations are a necessary condition for equilibrium growth, and "...also provide a source of equilibrating pressure once the system has been knocked off its equilibrium path."[21]

Ferguson, on the other hand, assumes the labor force growth rate changes over time, and submits that by allowing for labor to be substituted for capital in production, a variable accelerator may be introduced explicitly into growth models. He also stresses the importance of expectations upon the stability of the equilibrium growth rate and argues that these in turn are determined largely by the nature of governmental economic management.[22]

Smithies points out that Solow's article assumes that the capital output ratio will remain constant.[23] This, he notes, is not in agreement with studies by Solomon Fabricant

21. "Innovations and Equilibrium Growth", Economic Journal Vol. LXVI (September, 1956), pp. 455-466.

22. "On Theories of Acceleration and Growth", Quarterly Journal of Economics, Vol. LXXXIV (February, 1960), pp. 78-99.

23. "Productivity, Real Wages and Economic Growth", Quarterly Journal of Economics, Vol. LXXXIV (May, 1960), pp. 189-205.

which indicates that over the period from 1880-1957, this ratio has been declining.[24] Consequently, he introduces variable real wages which may or may not rise in accord with increases in productivity of labor. Where they do not, he argues that as output increases at an assumed steady pace, and capital is also increased, more than proportionate increments of labor will be added; consequently, the productivity of capital will rise and the capital output ratio will decline.

Others, such as Eisner[25] and Green[26] have disagreed with Solow's verdict that the stability of the Harrod-Domar model is quite possible; but their arguments in turn have been criticized.[27] In general, although the assumptions regarding the many possible variables in the system, as well as their behaviour, have differed, it now seems to be well established that the Harrod-Domar model may represent stable growth and as such is therefore a useful structure on which to base further analysis.

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24. Basic Facts on Productivity Change, (New York: National Bureau of Economic Research, 1959).

25. "On Growth Models and the Neo-Classical Resurgence", Economic Journal, Vol. LXVIII (December, 1958), pp. 707-721.

26. "Growth Models, Capital and Stability," Economic Journal, Vol. LXX (March, 1960), pp. 57-73.

27. See R. Solow, "Is Factor Substitution a Crime and If So, How Bad? Reply to Professor Eisner", Economic Journal, Vol. LXIX (September, 1959), pp. 597-599; also J. Tobin, "Reply to Professor Eisner," Economic Journal, Vol. LXIX (September, 1959), pp. 599-600.

Economic Growth Based on Cyclical Analysis

Harrod's lagless model was primarily a representation of the path of equilibrium growth. However, by introducing Gn as determining the ceiling rate of possible growth, and the eventual need for replacement investment as the floor, he tied it in with cyclical theory as well. He did not pretend that his was a complete explanation of the trade cycle but felt that the model displayed "...a framework within which a detailed theory of the cycle should be worked out."[28]

There have been a number of other approaches where long run growth and the cycle have been treated together. Two of the more recent expositions are those Matthews and Duesenberry which will now be examined. Both use the acceleration principle in a modified form, but each in a different manner and at a different stage in their analysis.

(1) Matthews' Interaction of the Cycle and the Trend

Matthews, as mentioned earlier, uses a lagged form of the capital stock adjustment principle (with allowance for expectations) and the multiplier in his explanation of the business cycle. This former concept was seen to be a generalization of the acceleration principle, which states that investment decisions vary directly with the level of national income and inversely with the stock of capital in existence; i.e., $I(t) = aY(t-1) - bK(t)$. When it was combined with the multiplier, four possible growth patterns could result; namely, a rise in income to a new constant level, damped or anti-damped cycles,

28. Towards a Dynamic Economics, p. 116.

and explosive growth.

In addition to these relatively short run forces, he envisaged certain influences making for a long run upward trend in output. These were rising consumption and investment demand, which were in turn a result of population growth and improved productivity and technology.[29] His argument involves a number of assumptions: With relation to consumption, if the overall level of real income remains the same between two periods, but productivity rises so that employment falls and unemployment rises, then, those who have lost their jobs will, he assumes, treat it as temporary and thus will utilize their savings in an effort to maintain their consumption. Those who remain employed will presumably have higher incomes (due to the higher productivity), and if they treat this as permanent, they will increase their consumption. The total effect will be a rise in aggregate consumption.

On the investment side, his reasoning is that a population expansion, discovery of new natural resources, or technical progress willhave the effect of increasing the supply of labor and other resources available to combine with any new capital that may be created. Thus the greater will be the potential productivity of new capital, and in turn, the higher will be the prospective rate of return on investment, given the level of the real national income and the size of the existing capital stock. He adds however, that the reliability of

29. Matthews, pp. 233-234.

this stimulus to investment will depend on the whether there is an excess demand for labor to begin with and whether real wages are flexible.

If there is an excess demand for labor, although wages may be inflexible, as might occur if the economy were pressing against the ceiling at the top of a boom, then increased labor availability would encourage investment. He feels this would still hold if there is not a universal excess demand for labor, but instead, only a shortage of labor of particular skills. But if there were no excess demand for labor when wages are inflexible, then a further increase in labor will do nothing to encourage investment. Where wages are flexible, an increase in the supply of labor will cause wages to decline and will raise profit margins and the expected rate of profit on capital and thus encourage investment. However, in this case, the stimulus to demand may be offset to some extent by a decline in consumption if the marginal propensity to consume of wage earners who now have lower incomes is higher than that of those who are receiving the profits. This entire argument assumes that there is a class of active entrepreneurs ready to do investment when prospective returns appear sufficient.

Matthews envisages that these trend factors would interact with the capital-stock-adjustment-multiplier forces to produce various patterns of behaviour of national income. For example, if the capital-stock-adjustment-multiplier process were such as to produce not either cycles or an explosive rise in income, and there were no other disturbances or shocks (an extremely unlikely situation), then the trend forces, i.e.,

population and technological growth, could cause income and capital stock to increase at a steady pace, which, using Harrod's terminology, he refers to as the natural rate of growth. More likely he feels is the case where the capital-stock-adjustment-multiplier process, plus perhaps other random disturbances, produce fluctuations which will occur around the rising trend. And then

> If the natural rate of growth is high, the upward trend in demand will be strong, and boom will be long and powerful and slumps short and weak. If the natural rate of growth is low, booms will be feebler and slumps more severe.[30]

In his view then, the growth factors may thus influence the cycle.

If the trend factors were not strong enough to stimulate expansion by themselves but were sufficient to at least remove obstacles to expansion such as a shortage of labor or other resources, then he believed that if the capital-stock-adjustment-multiplier process were such as to produce either antidamped cycles or explosive growth, it would be sufficient to push the economy to a higher level of output in each boom that occurred; i.e., the cyclical forces would thus be sufficient to produce growth, and the trend factors would in this case be exercising primarily a permissive influence on growth rather than providing positive stimulus.

A fourth possibility is that when the trend forces are weak, the capital-stock-adjustment-multiplier process pro-

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30. Ibid., p. 235.

duces damped cycles or a rise in income to a new plateau, but in neither case is the rise sufficient to bring the economy to a full employment level of activity, even in the boom. In other words, there may be damped cycles which are kept going by disturbances or shocks, but their is no growth; stagnation occurs.

In summary, Matthews sees four possibilities: where the capital-stock-adjustment-multiplier interaction coupled with the trend forces produce steady non-fluctuating growth; where these two sets of forces produce cycles around a rising trend; where, although the trend forces are not sufficient to provide any real stimulus to growth, they nevertheless exercise a permissive influence, so that the capital-stock-adjustment-multiplier process, if it is such as to produce anti-damped cycles or explosive expansion of income, will result in progressively higher levels of income at the peak of each boom, i.e., growth may occur; and finally where cycles may occur without any growth.

It can be seen therefore, that insofar as the acceleration principle is included in the capital stock adjustment principle, it plays an important part in Matthews' model of economic growth.

(2) Duesenberry's Approach[31]

Instead of starting with an explanation of the business cycle using some variation of the multiplier-accelerator process and imposing on this certain trend factors such as

31. J.S. Duesenberry, Business Cycles and Economic Growth, (New York; McGraw-Hill, 1958).

population and technology, Duesenberry utilizes a multiplier-accelerator type concept to explain long run growth, and submits that cyclical fluctuations are the result of extraneous influences.

First, a look at his explanation of long run growth. His equation for investment, when simplified appears quite similar to Matthews' capital stock adjustment principle, or generalized acceleration principle. It is expressed as $I(t) = \alpha Y(t-1) + \beta K(t-1)$, where I and Y are investment and income as usual, K(t-1) is capital stock at the end of period t-1, while α and β are coefficients; α will be positive and β negative.

However, rather than considering only the demand side of investment and thus the factors which may affect the marginal efficiency of capital such as the level of income, the stock of capital, and expectations, he argues that the supply of funds is also an important determinant of investment. He maintains that the supply schedule as faced by any firm is a function of a number of variables such as profits, debt, depreciation allowances and retained earnings. He sees it as relatively inelastic, at least over certain ranges. As such it will reduce the effect on investment of any improvement (shifts to the right) in the marginal efficiency of investment schedule. As a result, α in the above equation will be smaller than the same coefficient in Matthews' capital stock adjustment principle. Duesenberry recognizes that a rise in income will likely increase profits as well and thus make additional funds available

internally, but he does not think that this will prevent α from being well below unity.

Duesenberry assumes consumption is a lagged function of the level of income. He also provides for consumption to vary directly with the level of capital stock. This assumes that when capital stock is increased, there will be larger aggregate profits (as distinguished from the rate of profit) which will in turn result in higher dividends. These dividends increase personal income and thus consumption. The equation for consumption therefore takes the form $C(t) = aY(t-1) + bK(t-1)$, where a and b are positive coefficients measuring the net effects of $Y(t-1)$ and $K(t-1)$ on $C(t)$.

Using these functions, he develops a second order difference equation,[32] which, like the equation of Samuelson's cyclical model, is capable of producing four patterns of behaviour -- convergence to stationary equilibrium, damped or anti-damped cycles, and explosive growth. However, he submits that the parameters of the system have been such as to produce the fourth alternative. As a result of introducing the supply of funds as a limitation on investment, he believes that the exponential growth rate produced is sufficiently moderate as to be realistic in the light of historical experience of the United States. Furthermore, it is not too high to be sustained over the long run. In this manner, he explains growth.

As for the fluctuations that have occurred, he con-

32. Ibid., pp. 196-197.

tends that these are not due to the capital-stock-adjustment-multiplier-type process, but rather are the result of causes extraneous to the main process of growth, which must be explained separately. As he states,

> The argument given ... indicates that the observed "cycles" cannot be accounted for by a mechanism which tends to produce the same sequence of events over and over again. The observed cycles differ not only in detail but also in the basic causal mechanism behind them...[33]

The factors which he pin-points as being the crucial ones in generating major depressions since the American Civil War are such as: the collapse of a speculative boom in a particular industry (railways for example), the collapse of a speculative boom based on rising prices in a wage-price spiral, financial panic, and housing slumps.

In brief, Duesenberry utilizes the accelerator concept, although in a greatly modified form in his explanation of growth. But he discards this principle when diagnosing the causes of the cyclical fluctuations in economic activity in the United States.

(3) Other Approaches

Arthur Smithies, in his discussion of cycles and trends, expresses an opinion similar to Duesenberry's regarding the value of the acceleration principle. He believes that the "...cyclical accelerator is open to serious question".[34] He contends that the main weakness of cyclical theories embodying this concept

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33. Ibid., pp. 334.

34. "Economic Fluctuations and Growth", Econometrica, Vol. XXV (January, 1957), p. 11.

is that they assume businessmen, although aware of fluctuations, "...will act blindly on the basis of extrapolations of cyclical rates of growth and decline".[35] The level of past and present profits are for him a more important determinant of the investment that occurs over the cycle. But, as has been stressed earlier in this paper, the acceleration principle may still be operating, along with many other factors, through changes in profits; thus, acceptance of the profits principle does not, by itself, necessarily mean a rejection of the accelerator.

For the long run, Smithies recognized the accelerator as containing an important truth; the faster is the rate of growth of income the greater will be the inducements to invest. His analysis brings him to a model very similar to the Harrod warranted rate of growth.

Other growth models vary in their use of the acceleration principle. Tinbergen and Kalecki for example do not use it.[36] Goodwin, on the other hand believes it to be a valuable concept.[37] As seen in the previous chapter, Hicks gave it a crucial place in his analysis of the cycle but explains growth by a rising trend of autonomous investment.[38] Higgins

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35. Ibid.

36. Tinbergen, "Statistical Testing..." also The Dynamics of Business Cycles; Kalecki, Theory of Economic Dynamics.

37. "Secular and Cyclical Aspects of the Multiplier and the Accelerator", Income Employment and Public Policy: in Honor of Alvin H. Hansen, (New York, Norton, 1948).

38. A Contribution...

allows for the interaction of cycles and trends, and in so doing, the accelerator, which plays an important part in his cyclical theory, becomes one of the factors which influences growth.[39] Hamberg bases his explanation of growth on the Harrod model and thus utilizes the accelerator.[40]

Summary

The preceding discussion has indicated that the use of the acceleration principle in growth models has followed a pattern similar to that of its use in business cycle theory since the mid-thirties. It was first applied in its rigid form; other writers modified it to allow for other considerations; and still others rejected it entirely.

In its rigid, unqualified state, as in the unlagged Harrod-Domar type model, the accelerator was employed as one of the two key determinants of the rate of growth required for full capacity utilization of capital. The growth rate so determined was thought to be basically unstable; i.e., if the actual growth rate should depart from this warranted rate, a cumulative divergence of the two rates would follow. Subsequent analysis by Baumol, Solow, and others has shown that when allowance is made for expectations and variations in the proportions in which the factors of production are combined, the warranted rate may be fairly stable. As such it then becomes a more realistic

39. "Interactions of Cycles and Trends", Economic Journal, Vol. LXV (December, 1955), pp. 594-614.

40. Economic Growth and Instability.

representation of actual growth.

More recently, in the growth models of such as Matthews, Duesenberry, the acceleration principle has been generalized or modified to allow for the effects of not only the rate of change of income but also for the availability of funds and excess capacity.

Other writers have explained growth without utilizing the accelerator in any form; instead, such factors as a rising trend of autonomous investment due to population growth and technological change have been employed.

From these observations may be drawn certain conclusions regarding the importance of the acceleration principle in economic theory. These will be discussed in the following chapter.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Throughout the foregoing chapters, various interpretations, qualifications, and applications of the acceleration principle have been demonstrated. In Chapter II, it was noted that the accelerator, as a rigid relationship between output and investment, may be considered as operating between any two consecutive stages of production. As has been outlined, it may also be applied to a single firm's investment actions, to the longer run growth and decline of an industry, and to the aggregate concepts of consumption and investment, or to national income and aggregate investment. Differences in the time lags which may occur between changes in output and investment do not materially affect these basic relationships.

A survey was carried out in Chapter III of the various factors which may alter, or in some cases eliminate, the acceleration effect on investment. These factors included the influence of surplus capacity, expectations, replacement investment, availability of funds, shortage of other factors of production, technological change, indivisibility of capital, and the extent of depreciation which limits the acceleration effect in the downswing. Consequently, one can not expect to find many examples of the rigid accelerator in actual practice, and definitely not a constant acceleration coefficient that exists widely or for any length of time. The results of statistical analyses were seen to support this view.

It has also been noted that a number of theorists have rejected the accelerator as an explanation of induced investment, and turned instead to the profits principle which states that investment will vary with the expected rate of profit per unit of capital. However, the latter concept, rather than being used as an alternative to the acceleration principle, should properly be considered to include the general notion of the acceleration effect. The formulations of investment equations by those writers who embrace the profits principle may easily be interpreted as comprehending the accelerator as well.

The role of the accelerator in business cycles has also been outlined. It was noted that initially the principle was offered (in non-mathematical terms) as either the sole explanation of the trade cycle or as one of the contributing factors, and it was subsequently combined with the multiplier to form a self-contained theory of the cycle, with a gradually increasing emphasis being placed on the mathematical approach. Samuelson, one of the pioneers in this field, in order to illustrate the basic nature of the interaction process, used the accelerator in its rigid, unqualified form. Later writers in this area such as Hicks, have taken into consideration one or two of the factors (for example, replacement investment and the limits of disinvestment in the downswing to depreciation) which were known to influence the acceleration effect. In this manner, the use of a non-linear accelerator which differed between the upswing and the downswing was initiated. The analysis becomes more

complex, but the results remain the same. Four possible patterns of behaviour may obtain--convergence to a stable equilibrium, damped or anti-damped cycles, and explosive growth.

In more recent models, the acceleration principle has been generalized as the capital stock adjustment principle. This concept, by allowing for the influence of the level of capital stock at the beginning of the period, is actually just a method of introducing one more qualification which may alter the accelerator, namely, surplus capacity. The formulation also recognizes the broader concept of the profits principle. When combined with the multiplier the four patterns of behaviour that may result are the same as in earlier models.

Statistical tests of the capital stock adjustment principle, although they have not yielded definitely conclusive results, nevertheless have been generally more acceptable than the tests of the rigid accelerator. This is as might be expected since the generalized principle allows for the influence of one or two of the qualifications that may affect the accelerator.

The use of the acceleration principle in growth models was seen in Chapter V to have followed a pattern somewhat similar to its use in models of the business cycle. That is, it was noted that the principle was initially utilized in its rigid, unqualified form, and subsequently some of the limitations which affect it have been introduced.

What can be concluded from these discussions of the acceleration principle? Several points are evident. First, the concept cannot be considered as a general theory of investment because it does not explain replacement expenditures nor does it provide a satisfactory explanation of autonomous investment. Nor can it be used as a general theory of induced investment. The qualifications which have been pointed out as altering or even eliminating the acceleration effect are themselves good reason for not accepting this concept as a complete explanation of induced investment. Empirical evidence has also shown that induced investment activity has certainly not always varied in constant proportion to the rate of change of output. Moreover there is the statistical difficulty of separating induced from total investment. The profits principle, whereby induced investment is determined by the expected rate of profit per unit of capital, is recognized as a more comprehensive explanation. The accelerator may in fact work through changes in profits; that is, an alteration in the actual rate of profit (which will in turn have an effect upon the expected rate of profit on future investments) may well be an intermediate step between a change in the rate of change of output and a subsequent alteration in investment.

Thus, one is led to the conclusion that the accelerator is best considered as a partial explanation of induced investment. In other words, a variation in the rate of change of output is only one reason, perhaps the superficial reason, why changes in investment may occur. Other causes will include all those factors which may alter the expected rate of profit such as changes in interest rates,

prices of capital goods, quality of raw materials, efficiency of labor, technology, or administration, as well as the more indeterminate items such as the pyschological make-up of individuals and the sociological attributes of the population.

Viewed in this manner, the acceleration principle becomes difficult to refute. For example, even if an alteration in the rate of output produces no change in investment, this is not proof that the accelerator is invalid; it may only mean that variations in interest rates, prices of capital goods and so on, have reduced profit expectations and thus have offset the acceleration effect. This view of the accelerator also makes it difficult to verify. To do so necessitates its isolation from these other influences. One writer has stated that it would be necessary for trained economists to associate themselves on a personal basis over an extended period of time with the key executives throughout the country, and in this way get behind the answers which may appear on questionnaires about the reasons determining investment activity.[1] However, this suggestion is impracticable on a nationwide scale. Thus although in some isolated cases the accelerator has been found to be a valid concept, whether its influence as a determinant of investment is widespread or not, must remain largely a matter of conjecture.[2]

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1. D.B. Armstrong, The Acceleration Principle, Doctoral Thesis (Montreal: McGill University, 1954), p. 155.

2. It might be noted that even the profits principle, although it has a wider application than the accelerator, is also difficult to verify statistically.

These difficulties of measuring the acceleration effect and assessing its importance relative to the many other factors which may affect investment indicate that as a working concept, along with the multiplier, its usefulness as a means of predicting the effects of, say, changes in government expenditure on national income, or of alterations in demand in one industry upon output and investment in others, is most limited.

But the doubt which exists as to the accelerator's general applicability as a partial theory of investment and its weaknesses as a tool of economic forecasting must not be allowed to obscure what has been one of its major values. This is that, where it has been recognized, it has served, along with the multiplier, as a starting framework for the analysis of cyclical fluctuations and economic growth based on real as opposed to monetary factors. Some theorists have of course developed their models without recognizing the acceleration principle, but this does not deny the usefulness of this concept as a point of reference around which a number of authors have been able to marshal their thoughts on cyclical and growth phenomena.

Rather than being considered as a partial theory of investment, the accelerator may be treated simply as a structural ratio which relates additional units of capital to the extra output which may be produced with such capital. As seen in Chapter V, this ratio is the marginal capital coefficient. Where autonomous investment is excluded (or its productivity is the same as that of induced investment), this proportion will be the same as the

acceleration coefficient.

On a practical basis, this ratio has been utilized to determine, in rough terms, the quantity of capital required to raise the output of economies, particularly underdeveloped economies, by a given amount. But even this application is fraught with problems. For example, when measuring the amount of capital required to produce a unit of steel, does one measure only the actual machinery, equipment, and plant, and assume that the necessary skilled labor will be available? Or does one include in the ratio the construction of technical schools where the necessary skills may be learned? Should the transportation facilities required to bring the raw materials to the plant and take the products to the market be counted as well? Does one evaluate output and capital in physical units or, as is more likely, are monetary units used to relate the unlike physical units? And if monetary units are utilized, does one count the original purchase price of equipment, its depreciated value, or its replacement price? Then again, is gross output (including depreciation) or net output to be counted? Also, how does one allow for the differences in attitudes to work and the energy put forth by workers, even where the capital equipment and training of the men is identical?[3]

3. For a more complete discussion of these and similar considerations see J. Tinbergen, The Design of Development (Baltimore: John Hopkins, 1958); C.P. Kindleberger, Economic Development: Principles, Problems, and Policies, (New York: Norton, 1959); V.V. Bhatt, "Capital Output

Supposing that these problems are resolved and the capital coefficients for various types of investments determined, these coefficients would not, by themselves, indicate to planners the type of industry in which to invest. Consideration would also have to be given to such matters as the availability of labor and other natural resources, what products the country requires at home and will be able to sell abroad, and the durability of the various types of capital which will in turn indicate their total contribution to output over the long run. In short then, even where the accelerator is considered as no more than a structural proportion, many other factors have to be considered when utilizing this concept in planning for economic development.

With these thoughts in mind, what then are the future prospects for the acceleration principle? As a partial theory of investment, it will probably continue to be recognized in some circles. But because by itself it is extremely mechanical, additional factors will continue to be introduced into analyses of cyclical fluctuations and growth trends. Thus, its importance in the eyes even of those who continue to recognize it will likely be reduced. But because it does seem to have merit both on a priori and some empirical grounds, it will probably not be discarded entirely.

Ratios of Certain Industries: A Comparative Study of Certain Countries", Review of Economic and Statistics, Vol. XXXVI (August, 1954), pp. 309-320.

Its future as a mere ratio of capital to output will probably be similar to its future as a partial theory of induced investment. That is, it will continue to be used to some extent as one of the factors which must be considered when formulating plans for expansion of national output; but as new knowledge is obtained about the many other influences which will affect output, it will tend to hold a less important place. This trend has already become apparent in the past five years.[4]

4. See Kindleberger, p. 42; and Tinbergen, The Design..., pp. 70-76.

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